# 1 Introduction

## 1.1 DEFINITIONS

*Surveying* is the art of determining the relative positions of different objects on the surface of the earth by measuring the horizontal distances between them, and by preparing a map to any suitable scale. Thus, in this discipline, the measurements are taken only in the horizontal plane.

*Levelling* is the art of determining the relative vertical distances of different points on the surface of the earth. Therefore, in levelling, the measurements are taken only in the vertical plane.

## 1.2 OBJECT OF SURVEYING

The aim of surveying is to prepare a map to show the relative positions of the objects on the surface of the earth. The map is drawn to some suitable scale. It shows the natural features of a country, such as towns, villages, roads, railways, rivers, etc. Maps may also include details of different engineering works, such as roads, railways, irrigation canals, etc.

## 1.3 USES OF SURVEYING

Surveying may be used for the following various applications.

1. To prepare a topographical map which shows the hills, valleys, rivers, villages, towns, forests, etc. of a country.
2. To prepare a cadastral map showing the boundaries of fields, houses and other properties.
3. To prepare an engineering map which shows the details of engineering works such as roads, railways, reservoirs, irrigation canals, etc.
4. To prepare a military map showing the road and railway communications with different parts of a country. Such a map also shows the different strategic points important for the defence of a country.
5. To prepare a contour map to determine the capacity of a reservoir and to find the best possible route for roads, railways, etc.
6. To prepare a geological map showing areas including underground resources.
7. To prepare an archeological map including places where ancient relics exist.

## 1.8 CLASSIFICATION OF SURVEYING

### A. Primary Classification

Surveying is primarily classified as under:

1. Plane surveying and
2. Geodetic surveying.

*1 Plane surveying* We know that the shape of the earth is spheroidal. Thus, the surface is obviously curved. But in plane surveying, the curvature of the earth is not taken into consideration. This is because plane surveying is conducted over a small area. So, the surface of the earth is considered as plane. In such surveying, a line joining any two points is considered to be straight. The triangle formed by any three points is considered as a plane triangle and the angles of the triangle are assumed to be plane angles. Plane surveying is conducted by state agencies like the Irrigation Department, Railway Department, etc. Plane surveying is done on an area of less than 250 km$^2$.

*2 Geodetic surveying* In geodetic surveying, the curvature of the earth is taken into consideration. It is extended over a large area. The line joining any two points is considered as a curved line. The triangle formed by any three points is considered to be spherical and the angles of the triangle are assumed to be spherical angles. Geodetic surveying is conducted by the Survey of India department, and is carried out over an area exceeding 250 km$^2$.

### B. Secondary Classification

1. Based on instruments:
   - (a) Chain surveying,
   - (b) Compass surveying,
   - (c) Plane table surveying,
   - (d) Theodolite surveying,
   - (e) Tacheometric surveying, and
   - (f) Photographic surveying.
2. Based on methods:
   - (a) Triangulation surveying, and
   - (b) Traverse surveying.
3. Based on object:
   - (a) Geological surveying,
   - (b) Mine surveying,
   - (c) Archaeological surveying, and
   - (d) Military surveying.
4. Based on nature of field:
   - (a) Land surveying,
   - (b) Marine surveying, and
   - (c) Astronomical surveying.

Again, land surveying is divided into the following classes:

(i) Topographical surveying, which is done to determine the natural features of a country.

(ii) Cadastral surveying, which is conducted in order to determine the boundaries of fields, estates, houses, etc.

(iii) City surveying, which is carried out to locate the premises, streets, water supply and sanitary systems, etc.

(iv) Engineering surveying, which is done to prepare detailed drawings of projects involving roads, railways, etc.

## 1.5 GENERAL PRINCIPLE OF SURVEYING

The general principles of surveying are:

1. To work from the whole to the part, and
2. To locate a new station by at least two measurements (linear or angular) from fixed reference points.

1. According to the first principle, the whole area is first enclosed by main stations (i.e. controlling stations) and main survey lines (i.e. controlling lines). The area is then divided into a number of parts by forming well conditioned triangles. A nearly equilateral triangle is considered to be the best well-conditioned triangle. The main survey lines are measured very accurately with a standard chain. Then the sides of the triangles are measured. The purpose of this process of working is to prevent accumulation of error. During this procedure, if there is any error in the measurement of any side of a triangle, then it will not affect the whole work. The error can always be detected and eliminated.

But, if the reverse process (i.e. from the part to the whole) is followed, then the minor errors in measurement will be magnified in the process of expansion and a stage will come when these errors will become absolutely uncontrollable.

2. According to the second principle, the new stations should always be fixed by at least two measurements (linear or angular) from fixed reference points. Linear measurements refer to horizontal distances measured by chain or tape. Angular measurements refer to the magnetic bearing or horizontal angle taken by a prismatic compass or theodolite.

In chain surveying, the positions of main stations and directions of main survey lines are fixed by tie lines and check lines.

## 1.6 METHODS OF LINEAR MEASUREMENT

The following methods are generally employed for linear measurements:

***1. By pacing or stepping*** For rough and speedy work, distances are measured by pacing, i.e. by counting the number of walking steps of a man. The walking step of a man is considered as 2.5 ft or 80 cm. This method is generally employed in the reconnaissance survey of any project.

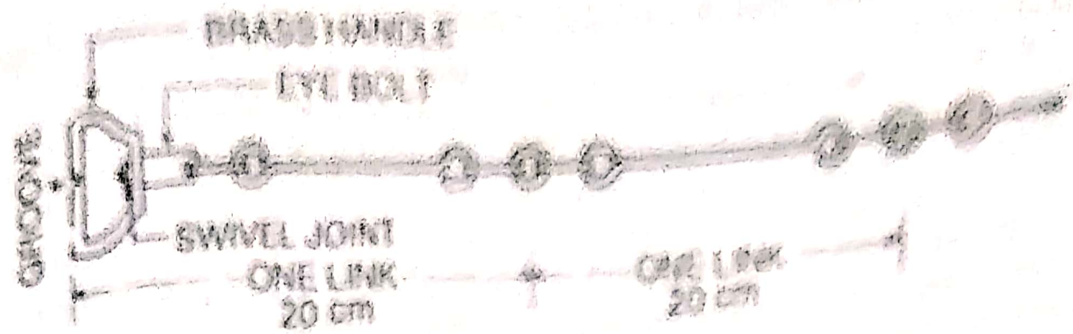



Fig. 1.2 Chain

(c) Engineers' chain,
(d) Gunter's chain, and
(e) Revenue chain.

*(a) Metric Chain:* Metric chains are available in lengths of 20 m and 30 m. The 20 m chain is divided into 100 links, each of 0.2 m. Tallies are provided at every 10 links (2 m). This chain is suitable for measuring distances along fairly level ground. The arrangement of tallies is shown in Fig. 1.3(a).

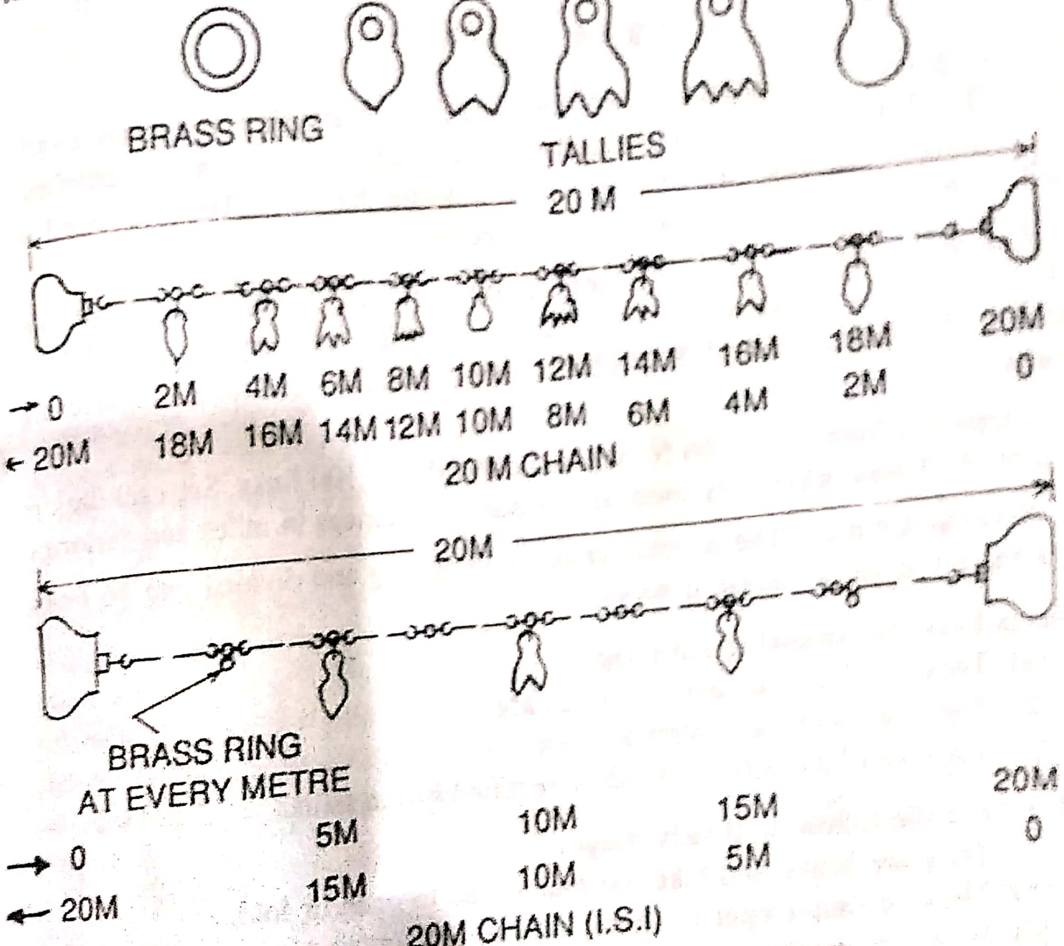



**Fig. 1.3** (a) 20 m Chain (b) 20 m Chain (ISI)

You may see from the arrangement of tallies that the central tally is round and that the other tallies have one, two, three or four teeth. So, each tooth may correspond to two different readings when considered from opposite ends. Therefore,

[illegible]

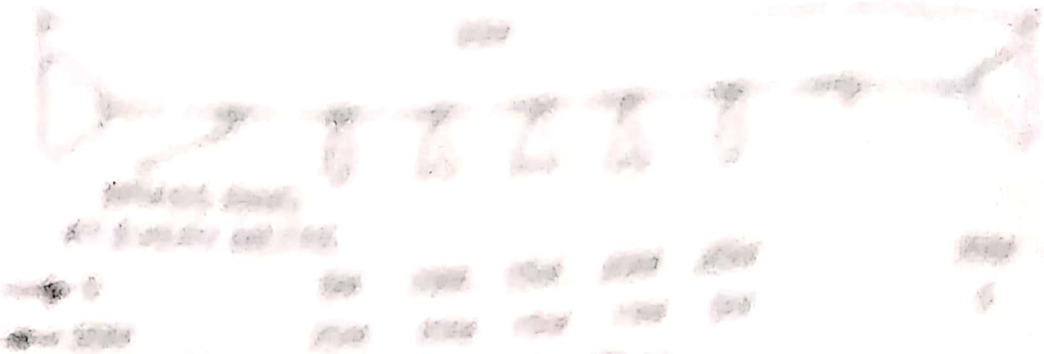


Fig. 1.6 Metric Chain

(b) Steel Band: It consists of a ribbon of steel of width 16 mm and of length 20 or 30 m. It has a brass handle at each end. It is graduated in metres, decimetres and centimetres on one side and has 0.2 m links on the other. The steel band is used in projects where more accuracy is required.

(c) Engineer's Chain: The engineer's chain is 100 ft long and is divided into 100 links. So, each link is of 1 ft. Tallies are provided at every 10 links (10 ft), the central tally being round. Such chains were previously used for all engineering works.

(d) Gunter's Chain: It is 66 ft long and divided into 100 links. So, each link is of 0.66 ft. It was previously used for measuring distances in miles and furlongs.

(e) Revenue Chain: The revenue chain is 33 ft long and divided into 16 links. It is mainly used in cadastral survey.

Chains have the following advantages:

(a) They can be read easily and quickly.
(b) They can withstand wear and tear.
(c) They can be easily repaired or rectified in the field.

They have the following disadvantages:

(a) They are heavy and take too much time to open or fold.
(b) They become longer or shorter due to continuous use.
(c) When the measurement is taken in suspension, the chain sags excessively.

Steel bands have the following advantages:

(a) They are very light and easy to open or fold.
(b) They maintain their standard length even after continuous use.
(c) When the measurement is taken in suspension, they sag slightly.

They have the following disadvantages:

(a) If handled carelessly, they break easily.
(b) They cannot be repaired in the field.
(c) They cannot be read easily.

3. **Tapes** The following are the different types of tapes:

(a) Cloth or linen tape,
(b) Metallic tape,
(c) Steel tape, and
(d) Invar tape.

*(a) Cloth or Linen Tape:* Such a tape is made of closely woven linen and is varnished to resist moisture. It is 15 mm wide and available in lengths of 10 and 15 m. This tape is generally used for measuring offsets and for ordinary works.

*(b) Metallic Tape:* When linen tape is reinforced with brass or copper wires to make it durable, then it is called a metallic tape. This tape is available in lengths of 15, 20 and 30 m. It is wound on a leather case with a brass handle at the end. It is commonly used for all survey works.

*(c) Steel Tape:* The steel tape is made of steel ribbon of width varying from 6 to 16 mm. The commonly available lengths are 10, 15, 20, 30 and 50 m. It is graduated in metres, decimetres and centimetres. It is not used in the field, but chiefly for standardising chains and for measurements in constructional works.

*(d) Invar Tape:* Invar tape is made of an alloy of steel (64 %) and nickel (36%). Its thermal coefficient is very low. Therefore, it is not affected by change of temperature. It is made in the form of a ribbon of width 6 mm and is available in lengths of 30, 50 and 100 m. It is used at places where maximum precision is required. It is generally used in the triangulation survey conducted by the Survey of India department.

**4. Arrows** Arrows are made of tempered steel wire of diameter 4 mm. One end of the arrow is bent into a ring of diameter 50 mm and the other end is pointed. Its overall length is 400 mm. Arrows are used for counting the number of chains while measuring a chain line (Fig. 1.5).

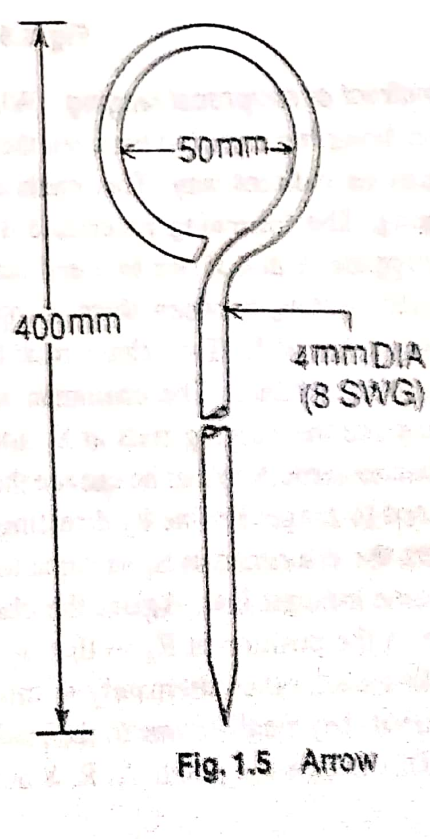


**Fig. 1.5** Arrow

## 1.8 RANGING

The process of establishing intermediate points on a straight line between two end points is known as ranging. Ranging must be done before a survey line is chained. Ranging may be done by direct observation by the naked eye or by line ranger or by theodolite. Generally, ranging is done by the naked eye with the help of three ranging rods.

Ranging may be of two kinds:

1. Direct, and
2. Indirect or reciprocal.

***1. Direct ranging*** When intermediate ranging rods are fixed on a straight line by direct observation from end stations, the process is known as direct ranging. Direct ranging is possible when the end stations are intervisible. The following procedure is adopted for direct ranging.

Assume that A and B are two end stations of a chain line, where two ranging rods are already fixed. Suppose it is required to fix a ranging rod at the intermediate point P on the chain line in such a way that the points A, P and B are in the same straight line. The surveyor stands about 2 m behind the ranging rod at A by looking towards the line AB. The assistant holds a ranging rod at P vertically at arm's length. The rod should be held lightly by the thumb and forefinger. Now, the surveyor directs the assistant to move the ranging rod to the left or right until the three ranging rods come exactly in the same straight line. To check the non-verticality of the rods, the surveyor bends down and looks through the bottom of the rods. The ranging will be perfect, when the three ranging rods coincide and appear as a single rod. When the surveyor is satisfied that the ranging is perfect, he signals the assistant to fix the ranging rod on the ground by waving both his hands up and down. Following the same procedure, the other ranging rods may be fixed on the line (Fig. 1.6).

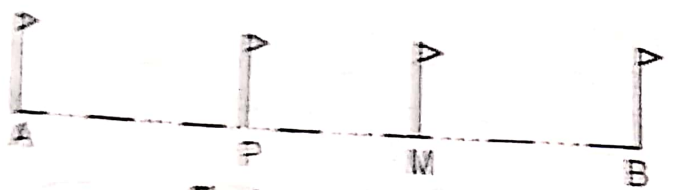



Fig. 1.6 Direct Ranging

***2. Indirect or reciprocal ranging*** When the end stations are not intervisible due to there being high ground between them, intermediate ranging rods are fixed on the line in an indirect way. This method is known as indirect ranging or reciprocal ranging. The following procedure is adopted for indirect ranging.

Suppose A and B are two end stations which are not intervisible due to high ground existing between them. Suppose it is required to fix intermediate points between A and B. Two chain men take up positions at $R_1$ and $S_1$ with ranging rods in their hands. The chainman at $R_1$ stands with his face towards B so that he can see the ranging rods at $S_1$ and B. Again, the chainman at $S_1$ stands with his face towards A so that he can see the ranging rods at $R_1$ and A. Then the chainmen proceed to range the line by directing each other alternately. The chainman at $R_1$ directs the chainman at $S_1$ to come to the position $S_2$ so that $R_1$, $S_2$ and B are in the same straight line. Again, the chainman at $S_2$ directs the chainman at $R_1$ to move to the position at $R_2$ so that $S_2$, $R_2$ and A are in the same straight line. By directing each other alternately in this manner, they change their positions every time until they finally come to the positions R and S, which are in the straight line AB. This means the points A, R, S and B are in the same straight line (Fig. 1.7).

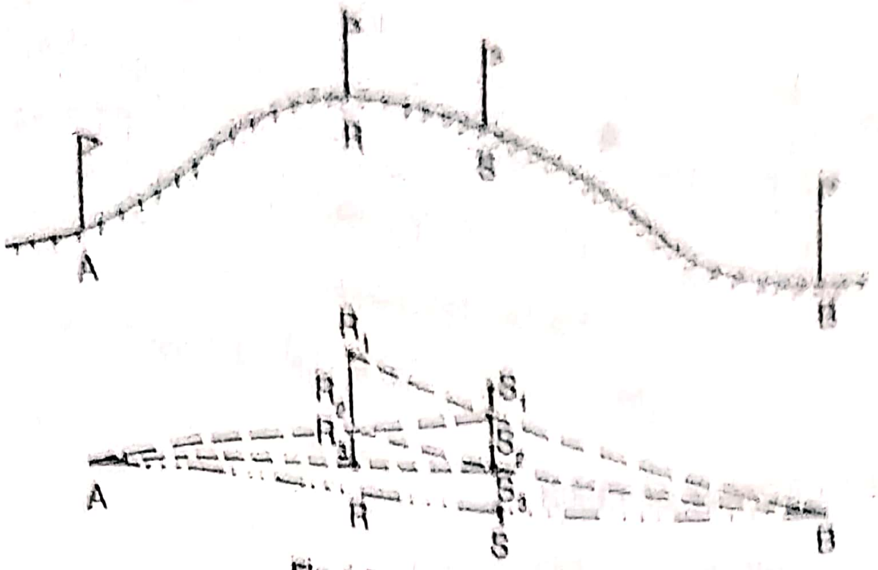



**Fig. 1.7** Reciprocal Ranging

## 1.9 UNFOLDING AND FOLDING A CHAIN

***1. Unfolding*** To open a chain, the strap is unfastened and the two brass handles are held in the left hand and the bunch is thrown forward with the right hand. Then one chainman stands at the starting station by holding one handle and another moves forward by holding the other handle until the chain is completely extended.

***2. Folding*** To fold the chain, a chainman should move forward by pulling the chain at the middle. Then the two halves of the chain will come side by side. After this, commencing from the central position of the chain, two pairs of links are taken at a time with the right hand and placed on the left hand alternately in both directions. Finally, the two brass handles will appear at the top. The bunch should be then fastened by the strap.

## 1.10 TESTING A CHAIN

Due to continuous use, a chain may be elongated or shortened. So, the chain should be tested and adjusted accordingly. If full adjustment is not possible, then the amount of shortening (known as 'too short') and elongation (known as 'too long') should be noted clearly for necessary correction applicable to the chain.

For testing the chain, a test gauge is established on a level platform with the help of a standard steel tape. The steel tape is standardised at 20°C and under a tension of 8 kg. The test gauge consists of two pegs having nails at the top and fixed on a level platform a required distance apart (say 20 or 30 m). The incorrect chain is fully stretched by pulling it under normal tension (say about 8 kg) along the test gauge. If the length of the chain does not tally with standard length, then an attempt should be made to rectify the error. Finally, the amount of elongation or shortening should be noted (Fig. 1.8).

The allowable error is about 2 mm per 1 m length of the chain. The overall length of the chain should be within the following permissible limits:

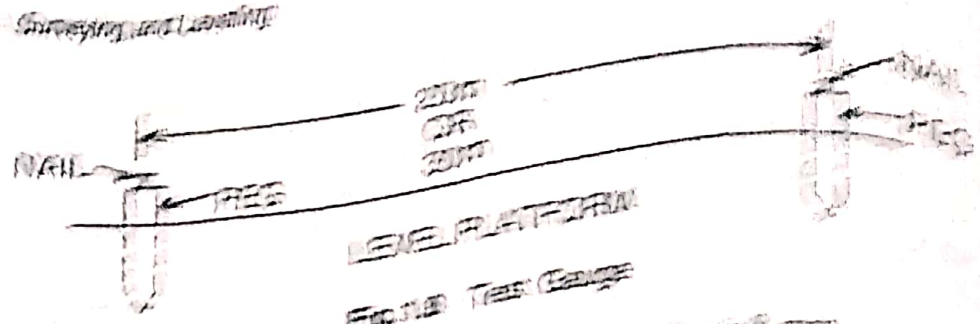



Fig. 1.10 Test Gauge

20 m chain: ± 5 mm  30 m chain: ± 8 mm

## 1.11 ADJUSTMENT OF CHAIN

Chains are adjusted in the following ways:

1. When the chain is too long, it is adjusted by:
   (a) Closing up the joints of the rings.
   (b) Hammering the elongated rings.
   (c) Replacing some old rings by new rings, and
   (d) Removing some of the rings.

2. When the chain is too short, it is adjusted by:
   (a) Straightening the bent links.
   (b) Opening the joints of the rings.
   (c) Replacing the old rings by some larger rings, and
   (d) Inserting new rings where necessary.

## 1.12 DEGREE OF ACCURACY IN CHAINING

The degree of accuracy in chaining is expressed as a ratio called the chaining ratio. The chaining ratio may be 1/1000, 1/2000, etc.

For example, if there is an error of 0.25 m during the measurement of a total length of 500 m.

$$\text{Chaining ratio} = \frac{0.25}{500} = \frac{25}{500 \times 100} = \frac{1}{2000}$$

Some permissible limits of error:

1. For measurement with steel band— $\frac{1}{2,000}$

2. For measurement with tested chain— $\frac{1}{1,000}$

3. In normal conditions— $\frac{1}{500}$

4. For rough work— $\frac{1}{250}$

1. compensating, or
2. cumulative.

**1. Compensating errors** Errors which may occur in both directions (i.e. both positive and negative) and which finally tend to compensate are known as compensating errors. These errors do not affect survey work seriously. They are proportional to $\sqrt{L}$, where $L$ is the length of the line. Such errors may be caused by:

(a) Incorrect holding of the chain.
(b) Horizontality and verticality of steps not being properly maintained during the stepping operation.
(c) Fractional parts of the chain or tape not being uniform throughout its length, and
(d) Inaccurate measurement of right angles with chain and tape.

**2. Cumulative errors** Errors which may occur in the same direction and which finally tend to accumulate are said to be cumulative. They seriously affect the accuracy of the work, and are proportional to the length of the line ($L$). The errors may be positive or negative.

*Positive errors* When the measured length is more than the actual length (i.e. when the chain is too short), the error is said to be positive. Such errors occur due to:

(a) The length of chain or tape being shorter than the standard length,
(b) Slope correction not being applied,
(c) Correction for sag not being made,
(d) Measurement being taken with faulty alignment, and
(e) Measurement being taken in high winds with the tape in suspension.

*Negative errors* When the measured length of the line is less than the actual length (i.e. when the chain is too long), the error is said to be negative. These errors occur when the length of the chain or tape is greater than the standard length due to the following reasons.

(a) The opening of ring joints,
(b) The applied pull being much greater than the standard pull,
(c) The temperature during measurement being much higher than the standard temperature,
(d) Wearing of connecting rings, and
(e) Elongation of the links due to heavy pull.

**3. Mistakes** Errors occurring due to the carelessness of the chainman are called "mistakes". The following are a few common mistakes:

(a) Displacement of arrows: Once an arrow is withdrawn from the ground during chaining, it may not be replaced in proper position, if required for to some reason.
(b) A full chain length may be omitted or added. This happens when arrows are lost or wrongly counted.

(c) [illegible]

(d) [illegible]

(e) [illegible]

(f) [illegible]

## 1.20 PRECAUTIONS AGAINST ERRORS AND MISTAKES

The following precautions should be taken to guard against errors and mistakes:

1. The point where the arrow is fixed on the ground should be marked with a cross (X).
2. The zero end of the chain or tape should be properly held.
3. During chaining, the number of arrows carried by the follower and leader should always tally with the total number of arrows taken.
4. While noting the measurement from the chain, the teeth of the tally should be verified with respect to the correct end.
5. The chainman should call the measurement loudly and distinctly and the surveyor should repeat them while booking.
6. Measurements should not be taken with the tape in suspension in high winds.
7. In stepping operations, horizontality and verticality should be properly maintained.
8. Ranging should be done accurately.
9. No measurement should be taken with the chain in suspension.
10. Care should be taken so that the chain is properly extended.

## 1.21 CHAIN AND TAPE CORRECTIONS

### A. Tape Correction

*1. Temperature correction ($C_t$)* This correction is necessary because the length of the tape or chain may be increased or decreased due to rise or fall of temperature during measurement. The correction is given by the expression

$$C_t = \alpha (T_m - T_0) L$$

where:

$C_t$ = correction for temperature, in metres
$\alpha$ = coefficient of thermal expansion
$T_m$ = temperature during measurement in degrees centigrade or celsius
$T_0$ = temperature at which the tape was standardised, in degrees centigrade or celsius
$L$ = length of tape, in metres

The sign of correction may be positive or negative according as $T_m$ is more or less than $T_0$.

When $\alpha$ for the steel tape is not given, it may be assumed as [illegible] per degree centigrade or celsius.

**2. Pull correction ($C_p$)** During measurement, the applied pull may be more or less than the pull at which the chain or tape was standardised. Due to elastic property of materials, the strain will vary according to the [illegible] pull, and hence necessary correction should be applied. This [illegible] by the expression

$$C_p = \frac{(P_m - P_0)L}{A \times E}$$

where $C_p$ = pull correction in metres

$P_m$ = pull applied during measurement, in kilograms

$P_0$ = pull at which the tape was standardised, in kilograms

$L$ = length of tape, in metre

$A$ = cross-sectional area of tape, in square centimetres

$E$ = modulus of elasticity (Young's modulus)

The sign of correction will be positive or negative according as $P_m$ is more or less than $P_0$.

When $E$ is not given, it may be assumed $2.1 \times 10^6$ kg/cm².

**3. Slope correction ($C_h$)** Slope correction is calculated as follows:

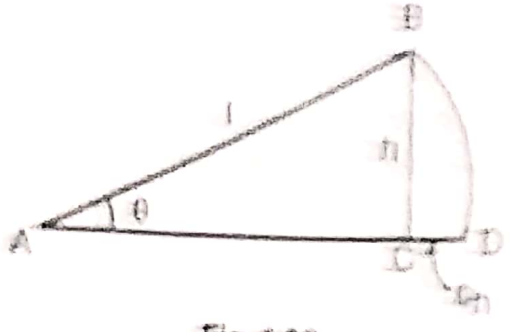



Fig. 1.22

$$C_h = l - \sqrt{l^2 - h^2} \quad \text{(exact)}$$

$$= l(1 - \cos\theta) \quad \text{(exact)}$$

$$= \frac{h^2}{2l} \quad \text{(approx.)}$$

This correction is always negative.

**4. Sag correction ($C_s$)** This correction is necessary when the measurement is taken with the tape in suspension (i.e. in the form of a catenary). It is given by the expression

$$C_s = \frac{L(wL)^2}{24n^2P_m^2}$$

when unit weight is given

and

$$C_s = \frac{LW^2}{24n^2P_m^2}$$

when total weight is given

where, $C_s$ = sag correction, in metres

$L$ = length of tape or chain, in metres
$w$ = weight of tape per unit length, in kilograms per metre
$W$ = total weight of tape, in kilograms
$n$ = number of spans
$P_m$ = pull applied during measurement, in kilograms

The sign of correction is always negative.

5. *Normal tension* ($P_n$) The tension at which the effect of pull is neutralised by the effect of sag is known as normal tension. At this tension, the elongation due to pull is balanced by the shortening due to sag. So, equating the expressions for correction for pull and sag, we have

$$\frac{(P_n - P_1)\,L}{AE} = \frac{L\,(wL)^2}{24\,P_n^2}\ \text{(considering } n = 1\text{)}$$

where, $P_n$ = normal pull or tension

Here the value of $P_n$ may be determined by trial, by forming an equation by putting the known values.

$$\frac{(P_n - P_1)\,L}{AE} = \frac{L\,(wL)^2}{24\,P_n^2}\ \text{(considering } n = 1\text{)}$$

or
$$\frac{(P_n - P_1)}{AE} = \frac{W^2}{24\,P_n^2}$$

or
$$(P_n - P_1)\,P_n^2 = \frac{W^2 AE}{24}$$

By substituting the values of $P_1$, $W$, $A$ and $E$, an equation will be obtained in the following form:

$$x P_n^3 \pm y P_n^2 \pm C = 0$$

Then, the value of $P_n$ is to be determined by satisfying the equation by trial and error.

## B. Chain Correction

**1. *Correction applied to incorrect length*** It is given by the expression

$$\text{True length of line (TL)} = \left(\frac{L'}{L}\right) \times \text{measured length (ML)}$$

where $L$ = standard or true length of chain
$L'$ = True length ± error
$= L \pm e$ ($e$ = error in chain or tape, i.e. when it is too long or too short)

Use the positive sign when the chain or tape is too long, the negative sign when it is too short.

**2. Correction of incorrect area** The correction to be applied in this case is given by the expression

$$\text{True area} = \left(\frac{L'}{L}\right)^2 \times \text{measured area}$$

**3. Hypotenusal allowance** This is explained in Sec. 1.15.
Hypotenusal allowance per tape = $L$ (sec $\theta$ − 1)

where $L$ = length of tape
$\theta$ = slope of the ground

This allowance is always added to the tape length.

## 1.22 WORKED OUT PROBLEMS ON CHAIN AND TAPE CORRECTIONS

***Problem 1*** The distance between two points, measured with a 20 m chain, was recorded as 327 m. It was afterwards found that the chain was 3 cm too long. What was the true distance between the points?

***Solution*** Given data:

True length of chain, $L$ = 20 m

Error in chain, $e$ = 3 cm = 0.03 m, too long

$L' = L + e = 20 + 0.03 = 20.03$ m

Measured length = 327 m

$$\text{True length of line} = \frac{L'}{L} \times \text{ML}$$

$$= \frac{20.03}{20} \times 327 = 327.49 \text{ m}$$

***Problem 2*** The distance between two stations was 1,200 m when measured with a 20 m chain. The same distance when measured with 30 m chain was found to be 1,195 m. If the 20 m chain was 0.05 m too long, what was the error in the 30 m chain?

***Solution*** Let us consider the 20 m chain.

$L$ = 20 m  $L'$ = 20 + 0.05 = 20.05 m

Measured length = 1,200 m

$$\text{True length of line} = \frac{20.05}{20} \times 1{,}200 = 1{,}203 \text{ m}$$

Let us now consider the 30 m chain.

$L$ = 30 m  $L'$ = ?

True length of line 1,203 m (as obtained from 20 m chain)
Measured length = 1,195 m.
From the relation

$$TL = \frac{L'}{L} \times ML$$

$$1{,}203 = \frac{L'}{30} \times 1{,}195$$

$$L' = \frac{1{,}203 \times 30}{1{,}195} = 30.20\,\text{m}$$

Now, $L'$ is greater than $L$. So, the chain is too long.
Amount of error, $e = 30.20 - 30 = +\,0.20$ m

***Problem 3*** A line was measured by a 20 m chain which was accurate before starting the day's work. After chaining 900 m, the chain was found to be 6 cm too long. After chaining a total distance of 1,575 m, the chain was found to be 14 cm too long. Find the true distance of the line.

***Solution*** First part:

$$L = 20\text{ m}$$

$$L' = 20 + \frac{0 + 0.06}{2} \text{ (considering mean elongation)}$$

$$= 20.03\text{ m}$$

$$ML = 900\text{ m}$$

$$TL = ?$$

$$TL = \frac{L'}{L} \times ML$$

$$= \frac{20.03}{20} \times 900 = 901.35\text{ m}$$

Second part:

$$L = 20\text{ m}$$

$$L' = 20 + \frac{0.06 + 0.14}{2} = 20.1\text{ m}$$

$$ML = 1{,}575 - 900 = 675\text{ m}$$

$$TL = \frac{20.1}{20} \times 675 = 678.375\text{ m}$$

True distance = 901.350 + 678.375 = 1,579.725 m

***Problem 4*** On a map drawn to a scale of 50 m to 1 cm, a surveyor measured the distance between two stations as 3,500 m. But it was found that by mistake he had used a scale of 100 m to 1 cm. Find the true distance between the stations.

***Solution*** First method:

As the surveyor used the scale of 100 m to 1 cm,

$$\text{Distance between stations on map} = \frac{3500}{100} = 35\text{ cm}$$

$$= \frac{30 \times (0.693)^2}{24 \times 2^2 \times (15)^2} = 0.00067 \text{ m } (-\text{ve})$$

$$\text{Total correction} = + 0.00396 + 0.00238 - 0.00067$$
$$= + 0.00567 \text{ m (too long)}$$

so $$L' = L + e = 30.00567$$

$$\text{True length} = \frac{30.00567}{30} \times 780 = 780.147 \text{ m}$$

**Problem 7** A 20-m steel tape was standardised on flat ground at a temperature of 20°C and under a pull of 15 kg. The tape was used in catenary at a temperature of 30°C and under a pull of $P$ kg. The cross-sectional area of the tape is 0.02 cm² and its total weight is 400 g. The Young's modulus and coefficient of expansion of steel are $2.1 \times 10^6$ kg/cm² and $11 \times 10^{-6}$ per °C respectively. Find the correct horizontal distance if $P$ is equal to 10 kg.

**Solution** Given data:

$L = 20$ m  
$T_0 = 20°C$  
$P_0 = 15$ kg  
$T_m = 30°C$  
$P = 10$ kg  
$A = 0.02$ cm²  
$\alpha = 11 \times 10^{-6}$ per °C  
$E = 2.1 \times 10^6$ kg/cm²  
$W = 400$ g $= 0.4$ kg  
$n = 1$

Here, applied pull $P = 10$ kg

(a) Temperature correction, $C_t = \alpha(T_m - T_0)\,L$

$$= 11 \times 10^{-6}\,(30 - 20)\,20$$
$$= 11 \times 10^{-6} \times 10 \times 20$$
$$= 0.00220 \text{ m } (+\text{ve})$$

(b) Pull correction, $C_p = \dfrac{(P - P_0)\,L}{A \times E}$

$$= \frac{(10 - 15)\,20}{0.02 \times 2.1 \times 10^6}$$
$$= -\frac{5 \times 20}{0.02 \times 2.1 \times 10^6}$$
$$= -0.00238 \text{ m } (-\text{ve})$$

(c) Sag correction, $C_s = \dfrac{L\,W^2}{24\,n^2\,P^2}$ $(n = 1)$

$$= \frac{20 \times (0.4)^2}{24 \times (10)^2} = 0.00133 \text{ m } (-\text{ve})$$

Total correction $= +0.00220 - 0.00238 - 0.00133 = -0.00151$ m

Correct horizontal distance $= 20 - 0.00151 = 19.99849$ m

***Problem 8*** A 30 m steel tape was standardised at a temperature of 20°C and under a pull 5 kg. The tape was used in catenary at a temperature of 25°C and under a pull of $P$ kg. The cross-sectional area of the tape is 0.02 cm², its weight per unit length is 22 g/m, Young's modulus = $2 \times 10^{-6}$ kg/cm², $\alpha = 11 \times 10^{-6}$ per °C. Find the correct horizontal distance, if $P$ is equal to (i) 5 kg, and (ii) 11 kg.

(WBSC 1986)

***Solution*** Given data:

$L = 30$ m

$T_0 = 20°C$

$P_0 = 5$ kg

$T_m = 25°C$

$P$ = (i) 5 kg (ii) 11 kg

$n = 1$

$A = 0.02$ cm²

$E = 2 \times 10^6$ kg/cm²

$\alpha = 11 \times 10^{-6}$ per °C

$W = 22$ g/m

Total weight $W = 22 \times 30$
= 660 g
= 0.66 kg

(a) When applied pull $P = 5$ kg:

(i) Temperature correction $C_t = \alpha(T_m - T_0)\,L$
$= 11 \times 10^{-6}\,(25 - 20)\,30$
$= 0.00165$ m (+ve)

(ii) Pull correction $= \dfrac{(P - P_0)\,L}{AE}$

$$= \frac{(5-5) \times 30}{0.02 \times 2 \times 10^6} = 0$$

(iii) Sag correction, $C_s = \dfrac{LW^2}{24n^2p^2} = \dfrac{30 \times (0.66)^2}{24 \times (5)^2}$ $(n = 1)$

$= +\,0.02178$ m (–ve)

Total correction = + 0.00165 – 0.02178 = – 0.02013 m
Correct horizontal distance = 30 – 0.02013 = 29.97987 m

(b) When applied pull $P = 11$ kg:

(i) Temperature correction $C_t = 0.00165$ m (+ve) as before.

(ii) Pull correction, $C_p = \dfrac{(P - P_0)\,L}{AE}$

$$= \frac{(11-5) \times 30}{0.02 \times 2 \times 10^6} = 0.0045 \text{ m (+ ve)}$$

(iii) Sag correction, $C_s = \dfrac{LW^2}{24n^2p^2}$ $(n = 1)$

$$= \frac{30 \times (0.66)^2}{24 \times (11)^2}$$

$= 0.00449$ m (– ve)

Total correction = + 0.00165 + 0.00450 − 0.00449

= + 0.00166 m

Correct horizontal distance = 30 + 0.00166

= 30.00166 m

**Problem 9** A steel tape was exactly 20 m long at 20°C when supported throughout its length under a pull of 5 kg. A line measured with this tape under a pull of 16 kg and at a mean temperature of 32°C, was found to be 680 m long. Assuming the tape is supported at every 20 m, find the true length of the line. Given that: (i) Cross-sectional area of tape = 0.03 cm², (ii) $E = 2.1 \times 10^6$ kg/cm², (iii) $\alpha = 11 \times 10^{-6}$ per °C, and (iv) weight of tape = 10 g/cc. (WBSC [illegible])

**Solution** Given data:

$L = 20$ m

$T_0 = 20°C$

$P_0 = 5$ kg

$T_m = 32°C$

$P_m = 16$ kg

$ML = 680$ m

$n = 1$

$A = 0.03$ cm²

$\alpha = 11 \times 10^{-6}$ per °C

$E = 2.1 \times 10^6$ cm²

Given weight = 10 g/cc

Total $W = 0.03 \times 20 \times 100 \times 10$

$= 600$ g $= 0.6$ kg

Correction per tape length:

(a) Temperature correction. $C_t = \alpha (T_m - T_0) L$

$= 11 \times 10^{-6} (32 - 20) \times 20$

$= 0.00264$ m (+ve)

(b) Pull correction. $C_T = \dfrac{(P_m - P_0) L}{AE}$

$= \dfrac{(16 - 5) \times 20}{0.03 \times 2.1 \times 10^6} = 0.00349$ m (+ve)

(c) Sag correction $C_s = \dfrac{L(W)^2}{24 P_m^2}$ $(n = 1)$

$= \dfrac{20 \times (0.6)^2}{24 \times (16)^2} = 0.00117$ m (−ve)

Total correction = + 0.00264 + 0.00349 − 0.00117

= + 0.00496 m

Actual length of tape, $L' = 20.00496$ m

True length of line $= \dfrac{L'}{L} \times ML$

$= \dfrac{20.00496}{20} \times 680 = 680.169$ m

# 2

# Chain Surveying

## 2.1 PRINCIPLE OF CHAIN SURVEYING

The principle of chain surveying is triangulation. This means that the area to be surveyed is divided into a number of small triangles which should be well conditioned. In chain surveying the sides of the triangles are measured directly on the field by chain or tape, and no angular measurements are taken. Here, the tie lines and check lines control the accuracy of work.

It should be noted that plotting triangles requires no angular measurements to be made, if the three sides are known.

Chain surveying is recommended when:

1. The ground surface is more or less level
2. A small area is to be surveyed
3. A small-scale map is to be prepared and
4. The formation of well-conditioned triangles is easy

Chain surveying is unsuitable when:

1. The area is crowded with many details
2. The area consists of too many undulations
3. The area is very large and
4. The formation of well-conditioned triangles becomes difficult due to obstacles

### A. Large-Scale and Small-Scale Maps

When 1 cm of a map represents a small distance, it is said to be a large-scale map.

For example,

$$1 \text{ cm} = 1 \text{ m} \quad \text{i.e.} \quad RF = \frac{1}{100}$$

When 1 cm of the map represents a large distance, it is called a small-scale map.

For example,

$$1 \text{ cm} = 100 \text{ m} \quad \text{i.e.} \quad RF = \frac{1}{10{,}000}$$

A map having an RF of less than 1/500 is considered to be large-scale. A map of RF more than 1/500 is said to be small-scale.

## 2.2 WELL-CONDITIONED AND ILL-CONDITIONED TRIANGLES

A triangle is said to be well-conditioned when no angle in it is less than 30° or greater than 120°. An equilateral triangle is considered to be the best-conditioned or ideal triangle (Figs 2.1(a) and (b)).

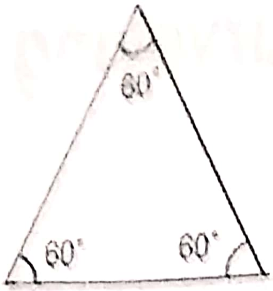



Fig. 2.1(a) Ideal Triangle

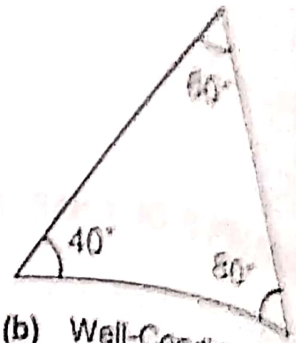



Fig. 2.1(b) Well-Conditioned Triangle

Well-conditioned triangles are preferred because their apex points are very sharp and can be located by a single 'dot'. In such a case, there is no possibility of relative displacement of the plotted point.

A triangle in which an angle is less than 30° or more than 120° is said to be ill-conditioned (Fig. 2.1(c)).

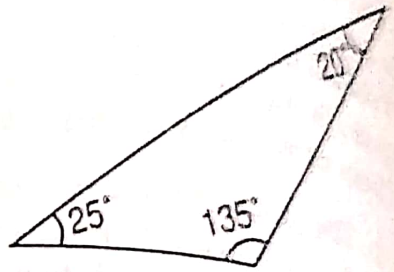



Fig. 2.1(c) Ill-Conditioned Triangle

Ill-conditioned triangles are not used in chain surveying. This is because their apex points are not sharp and well defined, which is why a slight displacement of these points may cause considerable error in plotting.

## 2.3 RECONNAISSANCE SURVEY AND INDEX SKETCH

Before the commencement of any survey work, the area to be surveyed is thoroughly examined by the surveyor, who then thinks about the possible arrangement of the framework of survey. This primary investigation of the area is termed as reconnaissance survey or reconnoitre.

During reconnaissance survey, the surveyor should walk over the area and note the various obstacles and whether or not the selected stations are intervisible. The main stations should be so selected that they enclose the whole area. The surveyor should also take care that the triangles formed are well-conditioned. He should note the various objects which are to be located.

The neat hand sketch of the area which is prepared during reconnaissance survey is known as the 'index sketch' or 'key plan'. The index sketch shows the skeleton of the survey work. It indicates the main survey stations, sub-stations, tie stations, base line, arrangement for framework of triangles and the approximate positions of different objects. This sketch is an important document for the surveyor and for the person who will plot the map. It should be attached to the starting page of the field book (Fig. 2.2).

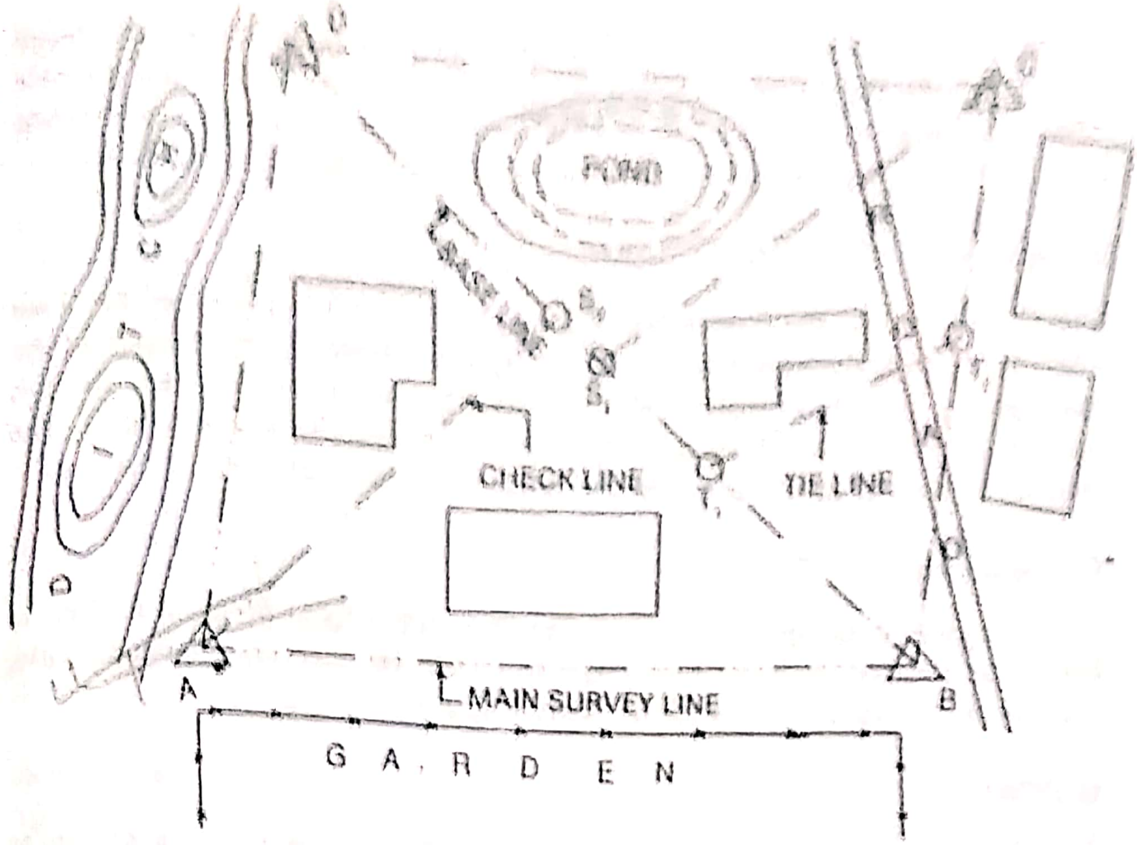



Fig. 2.2 Index Sketch

## 2.4 DEFINITIONS AND ILLUSTRATIONS

### A. Survey Stations

Survey stations are the points at the beginning and the end of a chain line. They may also occur at any convenient points on the chain line. Such stations may be:

1. Main stations
2. Subsidiary stations and
3. Tie stations

***1. Main stations*** Stations taken along the boundary of an area as controlling points are known as 'main stations'. The lines joining the main stations are called 'main survey lines'. The main survey lines should cover the whole area to be surveyed. The main stations are denoted by 'Δ' with letters A, B, C, D, etc. The chain lines are denoted by "— ... — ... — ... — ... —".

***2. Subsidiary stations*** Stations which are on the main survey lines or any other survey lines are known as "subsidiary stations". These stations are taken to run subsidiary lines for dividing the area into triangles, for checking the accuracy of triangles and for locating interior details. These stations are denoted by '⊙' with letters $S_1$, $S_2$, $S_3$, etc.

***3. Tie stations*** These are also subsidiary stations taken on the main survey lines. Lines joining the tie stations are known as tie lines. Tie lines are mainly taken to fix the directions of adjacent sides of the chain survey map. These are also taken to form 'chain angles' in chain traversing, when triangulation is not possible (chain angles are described in Chapter 3). Sometimes tie lines are taken to locate interior details. The stations are denoted by '⊙' with letters $T_1$, $T_2$, $T_3$, etc.

### B. Base Line

The line on which the framework of the survey is built is known as the 'base line'. It is the most important line of the survey. Generally, the longest of the main survey lines is considered the base line. This line should be taken through fairly level ground, and should be measured very carefully and accurately. The magnetic bearings of the base line are taken to fix the north line of the map.

### C. Check Line

The line joining the apex point of a triangle to some fixed point on its base is known as the 'check line'. It is taken to check the accuracy of the triangle. Sometimes this line helps to locate interior details.

### D. Offset

The lateral measurement taken from an object to the chain line is known as 'offset'. Offsets are taken to locate objects with reference to the chain line. They may be of two kinds—perpendicular and oblique.

***1. Perpendicular offsets*** When the lateral measurements are taken perpendicular to the chain line, they are known as perpendicular offsets (Fig. 2.3).

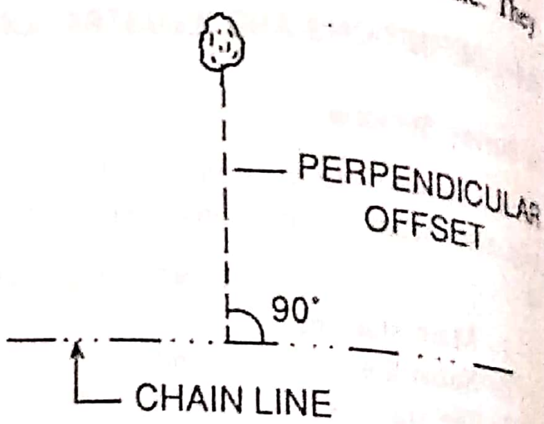



**Fig. 2.3**

Perpendicular offsets may be taken in the following ways:

(a) By setting a perpendicular by swinging a tape from the object to the chain line. The point of minimum reading on the tape will be the base of the perpendicular (Fig. 2.4).

(b) By setting a right angle in the ratio 3 : 4 : 5 (Fig. 2.5).

(c) By setting a right angle with the help of builder's square or tri-square (Fig. 2.6).

(d) By setting a right angle by cross-staff or optical square.

***2. Oblique offsets*** Any offset not perpendicular to the chain line is said to be oblique. Oblique offsets are taken when the objects are at a long distance from the chain line or when it is not possible to set up a right angle due to some difficulties. Such offsets are taken in the following manner.

Suppose AB is a chain line and p is the corner of a building. Two points 'a' and 'b' are taken on the chain line. The chainages of 'a' and 'b' are noted. The

6. The main survey lines should pass close to the boundary line of the area to be surveyed.
7. The survey lines should be taken close to the objects so that they can be located by short offsets.
8. The tie stations should be suitably selected to fix the directions of subsidiary lines.
9. The subsidiary stations should be suitably selected for taking check lines.
10. Stations should be so selected that obstacles to chaining are avoided as far as possible.
11. The survey lines should not be very close to main roads, as survey work may then be interrupted by traffic.

## 2.6 EQUIPMENTS FOR CHAIN SURVEY

The following equipments are required for conducting chain survey:

1. Metric chain (20 m) = 1 no.
2. Arrows = 10 nos
3. Metallic tape (15 m) = 1 no.
4. Ranging rods = 3 nos
5. Offset rod = 1 no.
6. Clinometer = 1 no.
7. Plumb bob with thread = 1 no.
8. Cross staff or optical square = 1 no.
9. Prismatic compass with stand = 1 no.
10. Wooden pegs = 10 nos
11. Mallet = 1 no.
12. Field book = 1 no.
13. Good pencil = 1 no.
14. Pen knife = 1 no.
15. Eraser (rubber) = 1 no.

## 2.7 THE FIELD BOOK

The notebook in which field measurements are noted is known as the 'field book'. The size of the field book is 20 cm × 12 cm and it opens lengthwise. Field books may be of two types:

1. Single-line, and
2. Double-line.

***1. Single-line field book*** In this type of field book, a single red line is drawn through the middle of each page. This line represents the chain line, and the chainages are written on it. The offsets are recorded, with sketches, to the left or right of the chain line. The recording of the field book is started from the last page and continued towards the first page. The main stations are marked by '$\Delta$' and subsidiary stations or tie stations are by '⨀' (Fig. 2.16).

**2. Double-line field book** In this type of field book, two red lines, 1.5 cm apart, are drawn through the middle of each page. This column represents the chain line, and the chainages are written in it. The offsets are recorded, with sketches, to the left or right of this column. The recording is begun from the last page and continued towards the first. The main stations are marked by '$\triangle$' and subsidiary or tie stations by '$\odot$' (Fig. 2.17). This type of field book is commonly used.

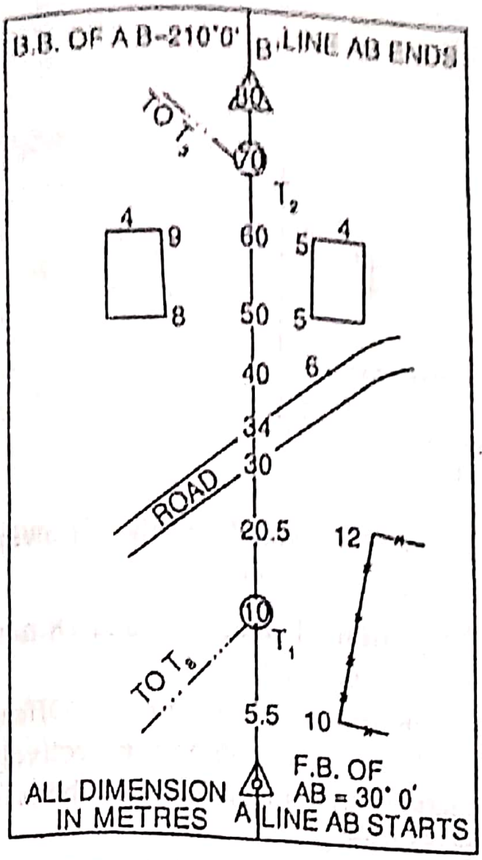



**Fig. 2.16** Single-Line Field Book

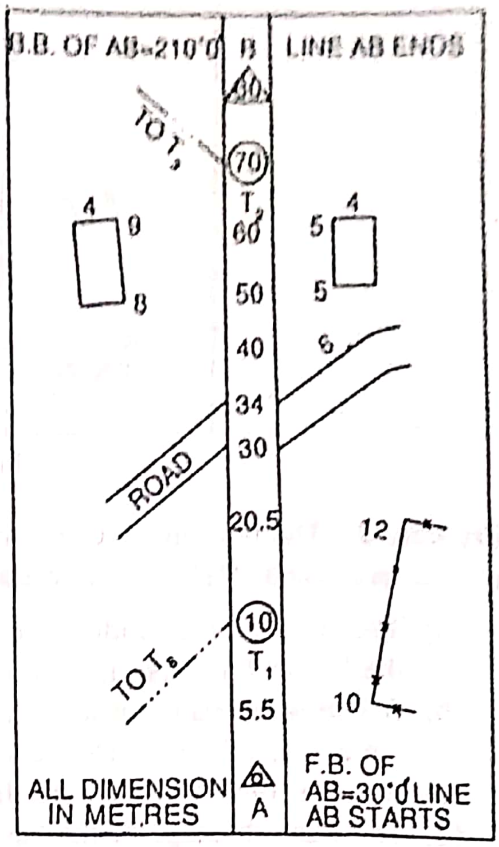



**Fig. 2.17** Double-Line Field Book

## A. Problems on Entering Records in Field Book

*Problem 1* While measuring a chain line AB, the following offsets are taken. How would you enter the field book?

(a) A telegraph post is 10 m perpendicularly from chainage 2.5 m to the right of the chain line.

(b) A road crosses obliquely from left to right at chainage 10 m and 14 m. Perpendicular offsets are 2 m and 3 m to the side of the road from chainage 5 m and 20 m respectively.

(c) A tube-well is 5 m perpendicularly from chainage 30 m to the left of the chain line.

(d) Total chainage of AB is 45 m.

**Problem 5** Enter the field book according to the following field notes:

(a) Chainage of line AB is 61.5 m

(b) The offsets to the pond to the left of chain line are as follows:

Chainage —10, 15, 20, 25, 30 m

Offset —16, 12, 10, 14, 20 m

(c) The offsets to the river at the right of the chain line are

Chainage—5, 25, 40, 60 m

Offset —13, 17, 19, 19.5 m

**Solution**

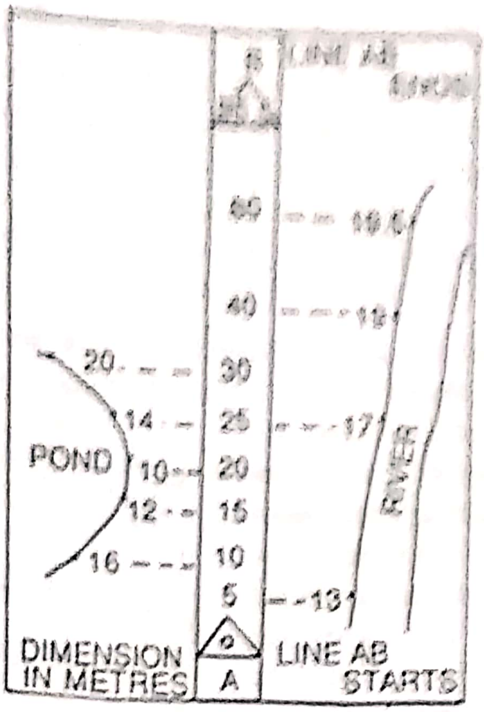



**Fig. P.2.5**

## B. Precautions to be Taken While Entering the Field Book

1. All measurements should be noted as soon as they are taken.
2. Each chain line should be recorded on a separate page. Normally it should start from the bottom of one page and end on the top of another. No line should be started from any intermediate position.
3. Over-writing should be avoided.
4. Figures and hand-writing should be neat and legible.
5. Index-sketch, object-sketch and notes should be clear.
6. Reference sketches should be given in the field book, so that the station can be located when required.
7. The field book should be entered in pencil and not in ink.
8. If an entry is incorrect or a page damaged, cancel the page and start the entry from a new one.
9. Erasing a sketch, measurement or note should be avoided.
10. The surveyor should face the direction of chaining so that the left-hand and right-hand objects can be recorded without any confusion.
11. The field-book should be carefully preserved.

# Compass Traversing

## 3.1 INTRODUCTION AND PURPOSE

In chain surveying, the area to be surveyed is divided into a number of triangles. This method is suitable for fairly level ground covering small areas. But when the area is large, undulating and crowded with many details, triangulation (which is the principle of chain survey) is not possible. In such an area, the method of traversing is adopted.

In traversing, the framework consists of a number of connected lines. The lengths are measured by chain or tape and the directions identified by angle measuring instruments. In one of the methods, the angle measuring instrument used is the compass. Hence, the process is known as compass traversing.

Note: Consideration of the traverse in an anticlockwise direction is always conventional in running the survey lines.

## 3.2 DEFINITIONS

***1. True meridian*** The line or plane passing through the geographical north pole, geographical south pole and any point on the surface of the earth, is known as the 'true meridian' or 'geographical meridian'. The true meridian at a station is constant. The true meridians passing through different points on the earth's surface are not parallel, but converge towards the poles. But for surveys in small areas, the true meridians passing through different points are assumed parallel.

The angle between the true meridian and a line is known as 'true bearing' of the line. It is also known as the 'azimuth' (Fig. 3.1).

***2. Magnetic meridian*** When a magnetic needle is suspended freely and balanced properly, unaffected by magnetic substances, it indicates a direction. This direction is known as the 'magnetic meridian'.

The angle between the magnetic meridian and a line is known as the 'magnetic bearing' or simply the 'bearing' of the line (Fig. 3.1).

***3. Arbitrary meridian*** Sometimes for the survey of a small area, a convenient direction is assumed as a meridian, known as the 'arbitrary meridian'. Sometimes the starting line of a survey is taken as the arbitrary meridian.

The angle between the arbitrary meridian and a line is known as the 'arbitrary bearing' of the line.

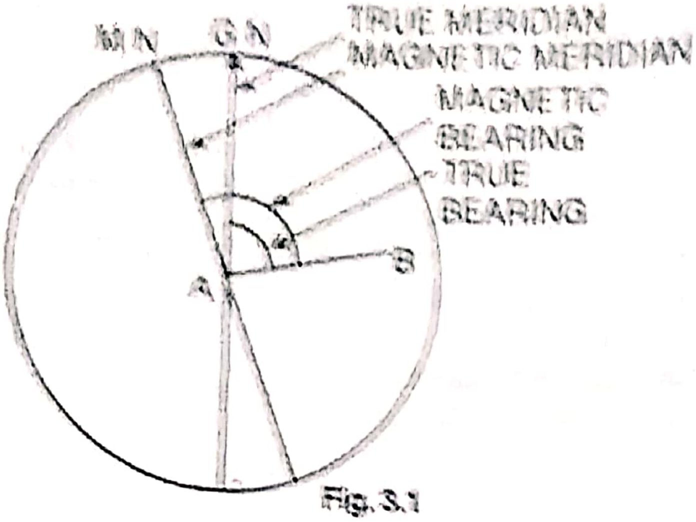



Fig. 3.1

*4. Grid meridian* Sometimes, for preparing a map some state agencies assume several lines parallel to the true meridian for a particular zone. These lines are termed as 'grid lines' and the central line the 'grid meridian'. The bearing of a line with respect to the grid meridian is known as the 'grid bearing' of the line.

*5. Designation of magnetic bearing* Magnetic bearings are designated by two systems:

(i) Whole circle bearing (WCB), and
(ii) Quadrantal bearing (QB).

*(a) Whole Circle Bearing (WCB)* The magnetic bearing of a line measured clockwise from the north pole towards the line, is known as the 'whole circle bearing, of that line. Such a bearing may have any value between 0° and 360°. The whole circle bearing of a line is obtained by prismatic compass (Fig. 3.2).

For example, in Fig. 3.2,

WCB of AB = $\theta_1$
WCB of AC = $\theta_2$
WCB of AD = $\theta_3$
WCB of AE = $\theta_4$

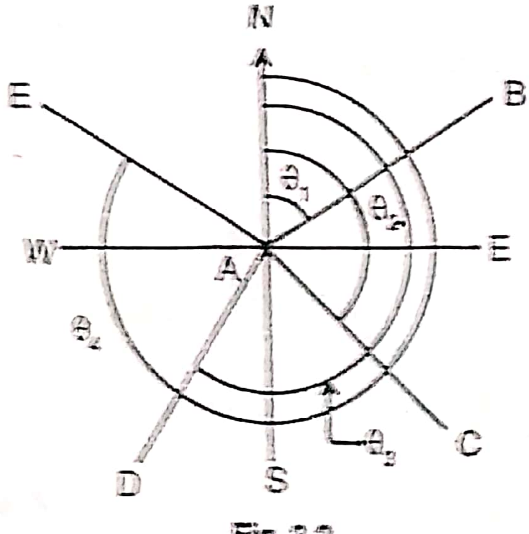



Fig. 3.2

*(b) Quadrantal Bearing (QB)* The magnetic bearing of a line measured clockwise or counterclockwise from the North Pole or South Pole (whichever is nearer the line) towards the East or West, is known as the 'quadrantal bearing' of the line. This system consists of four quadrants—NE, SE, SW and NW. The value of a quadrantal bearing lies between 0° and 90°, but the quadrants should always be mentioned. Quadrantal bearings are obtained by the surveyor's compass (Fig. 3.3).

For example, QB of AB = N$\theta_1$ E
QB of AC = S$\theta_2$ E
QB of AD = S$\theta_3$ W
QB of AE = N$\theta_4$ W

**6. Reduced bearing (RB)** [illegible]

The following table should be [illegible] for conversion of WCB to RB.

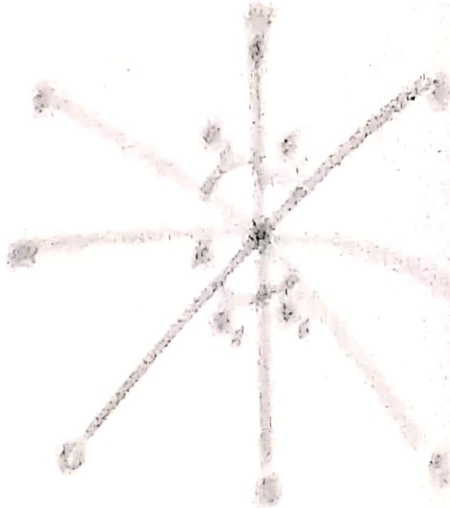


Fig. 3.3

| WCB between | Corresponding RB | Quadrant |
| --- | --- | --- |
| 0° and 90° | RB = WCB | NE |
| 90° and 180° | RB = 180° − WCB | SE |
| 180° and 270° | RB = WCB − 180° | SW |
| 270° and 360° | RB = 360° − WCB | NW |

***7. Fore and back bearing*** The bearing of a line measured in the direction of the progress of survey is called the 'fore bearing' (FB) of the line.

The bearing of a line measured in the direction opposite to the survey is called the 'back bearing' (BB) of the line (Fig. 3.4).

For example, in Fig. 3.4(a), FB of AB = $\theta$
BB of AB = $\theta_1$

In Fig. 3.4(b), FB of BA = $\theta$
BB of BA = $\theta_1$

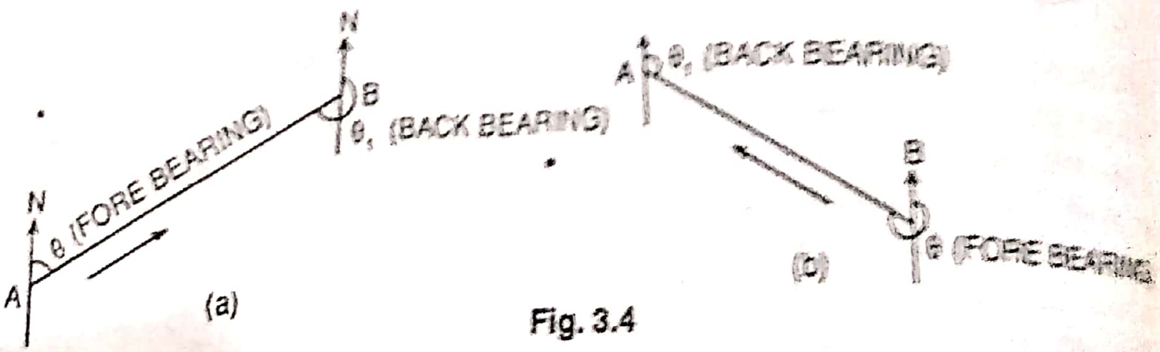



Fig. 3.4

Remember the following:

(a) In the WCB system, the difference between the FB and BB should be exactly 180°. Remember the following relation:

$$BB = FB \pm 180^\circ$$

Use the positive sign when FB is less than 180°, and the negative sign when it is more than 180°.

[illegible]

## 3.10 PROBLEMS ON WHOLE CIRCLE BEARINGS AND QUADRANTAL BEARINGS

*Problem 1* Convert the following WCBs to QBs.
(a) WCB of AB = [illegible]
(b) WCB of BC = [illegible]
(c) WCB of CD = [illegible]
(d) WCB of DE = [illegible]

*Solution*
(a) QB of AB = N [illegible] E
(b) QB of BC = 180°0' − [illegible] = [illegible] E
(c) QB of CD = [illegible] − 180°0' = [illegible] W
(d) QB of DE = 360°0' − [illegible] = [illegible] W

*Problem 2* Convert the following QBs to WCBs.
(a) QB of AB = S 36°30' W
(b) QB of BC = S 43°30' E
(c) QB of CD = N 26°45' E
(d) QB of DE = N 40°15' W

*Solution*
(a) WCB of AB = 180°0' + 36°30' = 216°30'
(b) WCB of BC = 180°0' − 43°30' = 136°30'
(c) WCB of CD = given QB = 26°45'
(d) WCB of DE = 360°0' − 40°15' = 319°45'

## 3.11 PROBLEMS ON FORE AND BACK BEARINGS

*Problem 1* The FBs of the following lines are given. Find the BBs.
(a) FB of AB = 310°30'

(b) FB of BC = 145°15'
(c) FB of CD = 210°30'
(d) FB of DE = 60°45'

**Solution:**

(a) BB of AB = 310°30' − 180°0' = 130°30'
(b) BB of BC = 145°15' + 180°0' = 325°15'
(c) BB of CD = 210°30' − 180°0' = 30°30'
(d) BB of DE = 60°45' + 180°0' = 240°45'

**Problem 2** FBs of the following lines are given. Find the BBs.

(a) FB of AB = S 30°30' E
(b) FB of BC = N 40°30' W
(c) FB of CD = S 60°15' W
(d) FB of DE = N 45°30' E

**Solution:**

(a) BB of AB = N 30°30' W
(b) BB of BC = S 40°30' E
(c) BB of CD = N 60°15' E
(d) BB of DE = S 45°30' W

**Problem 3** BBs of the following lines are given. Find the FBs.

(a) BB of AB = 40°30'
(b) BB of BC = 310°45'
(c) BB of CD = 145°45'
(d) BB of DE = 215°30'

**Solution:**

(a) FB of AB = 40°30' + 180°0' = 220°30'
(b) FB of BC = 310°45' − 180°0' = 130°45'
(c) FB of CD = 145°45' + 180°0' = 325°45'
(d) FB of DE = 215°30' − 180°0' = 35°30'

**Problem 4** BBs of the following lines are given. Find the FBs.

(a) BB of AB = N 30°30' W
(b) BB of BC = S 40°15' E
(c) BB of CD = N 60°45' E
(d) BB of DE = S 45°30' W

**Solution:**

(a) FB of AB = S 30°30' E
(b) FB of BC = N 40°15' W
(c) FB of CD = S 60°45' W
(d) FB of DE = N 45°30' E

*Problem 3* On an old map a line was drawn to a magnetic bearing of 3... when the declination was 3°30′ W. Find the present bearing of the line... declination is 4°15′ E.

*Solution*

True bearing = magnetic bearing – declination (west)

= 320°30′ – 3°30′ = 317°0′

The true bearing of a line is constant.

So, the present true bearing of the line is also 317° 0′

Magnetic bearing = true bearing – declination (east)

= 317°0′ – 4°15′ = 312°45′

*Problem 4* (a) The magnetic bearing of the sun at noon is 175°30′ from a st... Find the magnetic declination at that station.

(b) The magnetic bearing of the sun at noon is 5°30′ from other st... Find the magnetic declination at that station.

*Solution* (a) The sun is exactly on the true meridian at noon. Since the mag... bearing of the sun is 175°30′, it is towards the south pole of... meridian. Draw a true meridian and the sun place towards... south pole. Then set out an angle of 175°30′ anticlockwise... the sun to get the magnetic meridian.

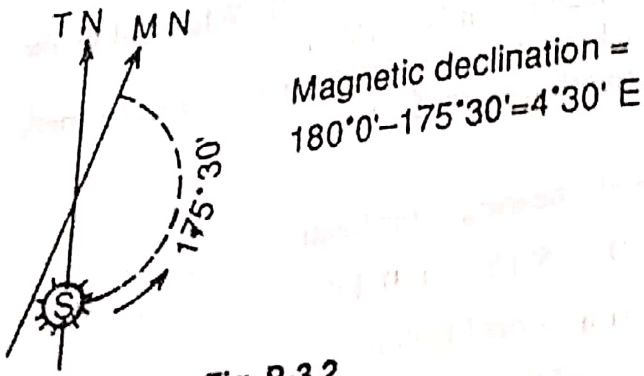



Fig. P-3.2

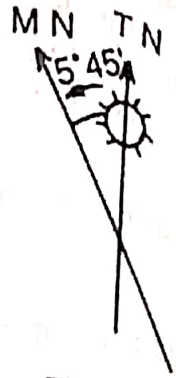



Fig. P-3.3

Thus Magnetic declination = 180°0′ – 175°30′ = 4°30′ E

(b) The sun is exactly on the true meridian at noon. Since the magnetic b... of the sun is 5°45′, it is towards the north pole of true meridian. Dr... sun towards the north pole of the true meridian. After that, set out a... of 5°45′ anticlockwise from the sun to get the magnetic meridian.

Thus Magnetic declination = 5°45′ W

## 3.13 PROBLEMS ON INCLUDED ANGLE

*Problem 1* The bearings of the lines OA, OB, OC, OD are 30°30′, 140°15′, ... and 310°30′, respectively. Find the angles ∠AOB, ∠BOC and ∠COD.

**Solution:**

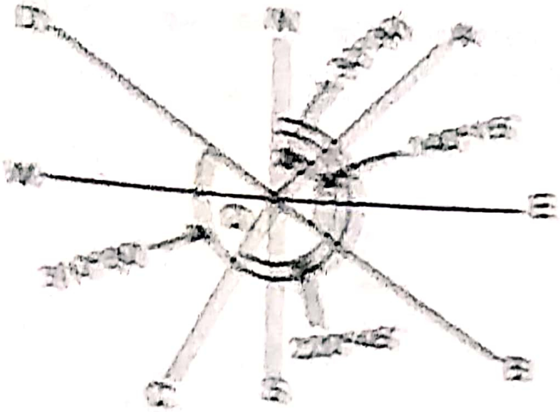



Fig. P-2.4

$\angle AOB$ = Bearing of OB − bearing of OA
= 140°15' − 30°30' = 109°45'

$\angle BOC$ = Bearing of OC − bearing of OB
= 220°45' − 140°15' = 80°30'

$\angle COD$ = Bearing of OD − bearing of OC
= 310°30' − 220°45' = 89°45'

**Problem 2** The fore bearings of the lines AB, BC, CD and DE, are 45°30', 120°15', 200°30' and 290°45', respectively. Find angles $\angle B$, $\angle C$ and $\angle D$.

**Solution:**

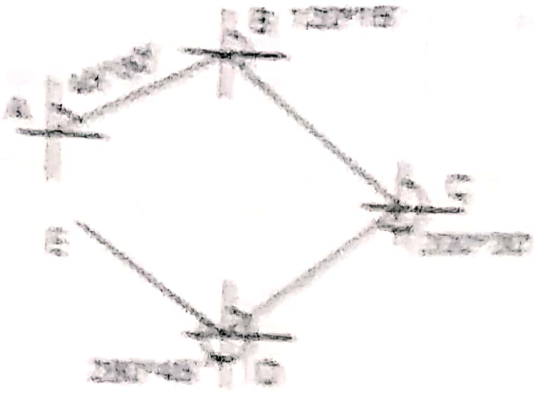



Fig. P-2.5

Interior $\angle B$ = BB of AB − FB of BC
= (45°30' + 180°00') − 120°15'
= 225°30' − 120°15' = 105°15'

Interior $\angle C$ = BB of BC − FB of CD
= (120°15' + 180°00') − 200°30'
= 300°15' − 200°30' = 99°45'

Exterior $\angle D$ = FB of DE − BB of CD
= 290°45' − (200°30' − 180°00')
= 290°45' − 20°30' = 270°15'

Interior $\angle D$ = 360°00' − 270°15' = 89°45'

**Problem 3** A traverse is done by three stations A, B and C in clockwise order

## 3.14 PROBLEMS ON LOCAL ATTRACTION

**Problem 1:** The following are the observed bearings of the lines of a traverse ABCDEA with a compass in a place where local attraction was suspected.

| Line | FB | BB |
|---|---|---|
| AB | 191°45' | 13°00' |
| BC | 39°30' | 222°30' |
| CD | 22°15' | 200°30' |
| DE | 242°45' | 62°45' |
| EA | 330°15' | 147°45' |

Find the correct bearings of the lines. (WBSC 1960)

**Solution** *First method—By calculating interior angles*

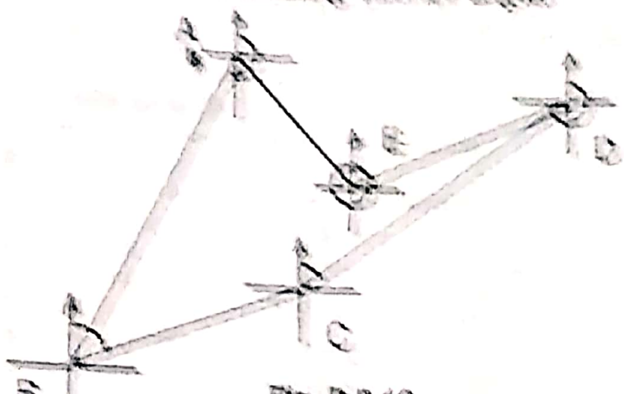



Fig. P-3.12

*(a) Calculation of interior angle*

Interior ∠A = FB of AB − BB of EA = 191°45' − 147°45' = 44°00'
Interior ∠B = FB of BC − BB of AB = 39°30' − 13°00' = 26°30'
Exterior ∠C = BB of BC − FB of CD = 222°30' − 22°15' = 200°15'
Interior ∠C = 360°00' − 200°15' = 159°45'
Interior ∠D = FB of DE − BB of CD
= 242°45' − 200°30' = 42°15'
Interior ∠E = FB of EA − BB of DE
= 330°15' − 62°45' = 267°30'

Sum of interior angles = 44°00' + 26°30' + 159°45' + 42°15' + 267°30'
= 540°00'

which is equal to (2N − 4) × 90° = 540°00'

So, the calculated angles are correct.

*(b) Calculation of corrected bearing*

The line DE is free from local attraction. So,

FB of DE = 242°45' (correct)
and FB of EA = 330°15' (correct)

[illegible]

The result is tabulated as follows

| Line | Corrected | |
|---|---|---|
| | FB | BB |
| AB | 194°15′ | 14°15′ |
| BC | 40°45′ | 220°45′ |
| CD | 20° 30′ | 200°30′ |
| DE | 242°45′ | 62°45′ |
| EA | 330°15′ | 150°15′ |

*Second method—Directly applying correction*

*Procedure* (a) On verifying the observed bearing it is found that the FB and BB of line DE differ by exactly 180°. So, the stations D and E are free from local attraction and the observed FB and BB are correct.

(b) The observed FB of EA is also correct.

(c) The actual BB of EA should be

$$330°15' - 180°0' = 150°15'$$

But the observed bearing is 147°45′.
So, a correction of (150°15′ – 147°45′) = + 2°30′ should be applied

(d) Correct FB of AB = 191°45′ + 2°30′ = 194°15′
Therefore, the actual correct BB of AB should be

$$194°15' - 180°00' = 14°15'$$

But Observed bearing = 13°0′

So, a correction of (14°15′ – 13°0′) = + 1°15′ should be ap
at B.

(e) Correct FB of BC = 39°30' + 1°15' = 40°45'

∴ Correct BB of BC should be = 40°45' + 180°0' = 220°45'

But Observed bearing of BC = 222°30'

So, a correction of

(220°45' − 222°30') = − 1°45' should be applied at C.

(f) Correct FB of CD = 22°15' − 1°45' = 20°30'

Therefore, the BB of CD should be

20°30' + 180°0' = 200°30'

which tallies with the observed BB of CD.

So, D is free from local attraction, which also tallies with the remark made at the beginning.

The result is tabulated as follows:

Table for Correction

| Line | Observed | | Correction | Correct | | Remarks |
|---|---|---|---|---|---|---|
| | FB | BB | | FB | BB | |
| AB | 191°45' | 13°00' | + 2°30' at A | 194°15' | 14°15' | |
| BC | 39°30' | 222°30' | + 1°15' at B | 40°45' | 220°45' | |
| CD | 22°15' | 200°30' | − 1°45' at C | 20°30' | 200°30' | |
| DE | 242°45' | 62°45' | 0° at D | 242°45' | 62°45' | Station D is free from local attraction |
| EA | 330°15' | 147°45' | 0° at E | 330°15' | 150°15' | Station E is also free from local attraction |

***Problem 2*** The following bearings were observed in traversing, with a compass, an area where local attraction was suspected. Find the amounts of local attraction at different stations, the correct bearings of lines and the included angles. Also draw a sketch of the plot if AB = 100 m, BC = 100 m and CD = 50 m and show in it all the included angles.

(WBSC 1989).

| Line | FB | BB |
|---|---|---|
| AB | 68°15' | 248°15' |
| BC | 148°45' | 326°15' |
| CD | 224°30' | 46°00' |
| DE | 217°15' | 38°15' |
| EA | 327°45' | 147°45' |

[illegible]

[illegible]

[illegible]

**Limits of closing error** The angular error of closure should not exceed [illegible] where N is the number of sides of the traverse.

$$\text{Relative closing error} = \frac{\text{amount of closing error}}{\text{perimeter of traverse}}$$

The value should not exceed [illegible]

## 3.18 SOURCES OF ERROR IN COMPASS

The following are the kinds of error which may occur while taking [illegible] a compass.

**1. Instrumental errors**

(a) The needle may not be perfectly straight and might not be [illegible]
(b) The pivot point may be eccentric.
(c) The graduations of the ring may not be uniform.
(d) The ring may not rotate freely on account of the pivot point [illegible] This may occur due to the head of the pivot being [illegible] careless handling.
(e) The sight vane may not be vertical.
(f) The horse hair may not be straight and vertical.

**2. Personal errors**

(a) The centring may not be done perfectly over the station.
(b) The graduated ring may not be levelled.
(c) The object might not be bisected properly.
(d) The readings may be taken or entered carelessly.
(e) The observer may be carrying magnetic substances.

**3. Other sources of error**

(a) There may be local attraction due to the presence of magnetic [illegible] near the station.

# 4

# Plane Table Surveying

## 4.1 PRINCIPLE

The principle of plane tabling is parallelism, meaning that the rays drawn from stations to objects on the paper are parallel to the lines from the stations to the objects on the ground. The relative positions of the objects on the ground are represented by their plotted positions on the paper and lie on the respective rays. The table is always placed at each of the successive stations parallel to the position it occupied at the starting station. Plane tabling is a graphical method of surveying. Here, the field work and plotting are done simultaneously and such survey does not involve the use of a field book.

Plane table survey is mainly suitable for filling interior details when traversing is done by theodolite. Sometimes traversing by plane table may also be done. But this survey is recommended for the work where great accuracy is not required. As the fitting and fixing arrangement of this instrument is not perfect, most accurate work cannot be expected.

## 4.2 ACCESSORIES OF PLANE TABLE

**1. The Plane Table** The plane table is a drawing board of size 750 mm × 600 mm made of well-seasoned wood like teak, pine, etc. The top surface of the table is well levelled. The bottom surface consists of a threaded circular plate for fixing the table on the tripod stand by a wing nut.

The plane table is meant for fixing a drawing sheet over it. The positions of the objects are located on this sheet by drawing rays and plotting to any suitable scale (Fig. 4.1).

**2. The Alidade** There are two types of alidade—plain and telescopic.

**(a) Plain Alidade** The plain alidade consists of a metal or wooden ruler of length about 50 cm. One of its edges is bevelled, and is known as the fiducial edge. It consists of two vanes at both ends which are hinged with the ruler. One is known as the 'object vane' and carries a horse hair; the other is called the 'sight vane' and is provided with a narrow slit (Fig. 4.2).

**(b) Telescopic Alidade** The telescopic alidade consists of a telescope meant for inclined sight or sighting distant objects clearly. This alidade has no vanes at the ends, but is provided with fiducial edge.

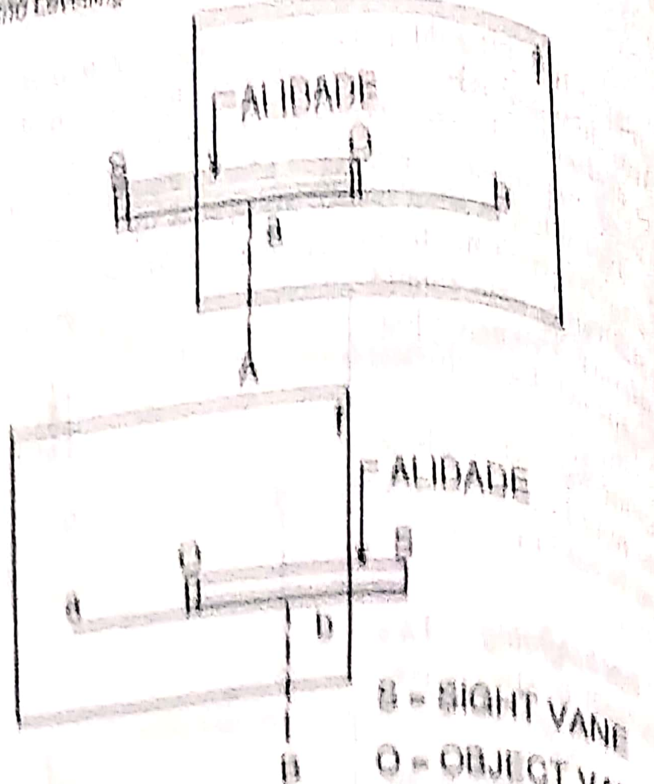



Fig. 4.9

corners and various positions on the table. The bubble is brought to the ... its run at every positions of the table by adjusting the legs.

**3. *Centring the table*** The drawing sheet is fixed on the table. A suitable ... is selected on the sheet to represent the station P on the ground. A pin ... fixed on this selected point. The upper pointed end of the U-fork is ... contact with the station pin and the plumb bob which is suspended from ... at the lower end is brought just over the station P by turning the table ... or anticlockwise or slightly adjusting the legs. This operation is called ... The table is then clamped. Care should be taken not to disturb the ...

**4. *Marking the north line*** The trough compass is placed on the right ... corner with its north end approximately towards the north. Then the table ... turned clockwise or anticlockwise so that the needle exactly coincides ... 0–0 mark. Now a line representing the north line is drawn through the ... compass. It should be ensured that the table is not turned.

**5. *Orientation*** When plane table survey is to be conducted by connecting ... stations, the orientation must be performed at every successive station ... done by magnetic needle or by the backsighting method. The backsighting ... is always preferred, because it is reliable. During orientation, it should be ... remembered that the requirements of centring, levelling, and orientation ... satisfied simultaneously.

## 4.5 METHODS OF PLANE TABLING

The following are the four methods of plane tabling:

1. Radiation
2. Intersection
3. Traversing and
4. Resection

**1. Radiation** This method is suitable for locating the objects from a single station. In this method, rays are drawn from the station to the objects, and the distances from the station to the objects are measured and plotted to any suitable scale along the respective rays.

*Procedure* (a) Suppose P is a station on the ground from where the objects A, B, C and D are visible.

(b) The plane table is set up over the station P. A drawing sheet is fixed on the table, which is then levelled and centered. A point *p* is selected on the sheet to represent the station P.

(c) The north line is marked on the right-hand top corner of the sheet with trough compass or circular box compass.

(d) With the alidade touching *p*, the ranging rods at A, B, C and D are bisected and the rays drawn.

(e) The distances PA, PB, PC, and PD are measured and plotted to any suitable scale to obtain the points a, b, c, and d, representing the objects A, B, C and D (Fig. 4.10), on paper.

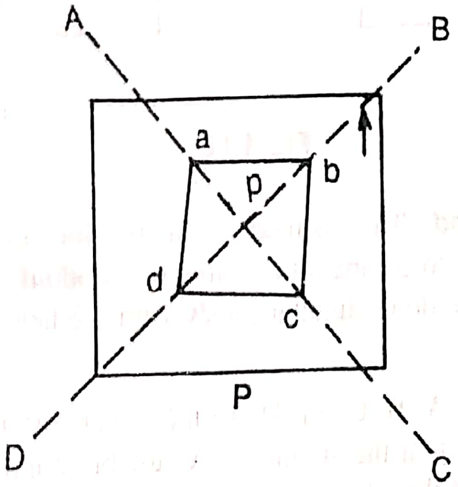



**Fig. 4.10**

**2. The Intersection method** This method is suitable for locating inaccessible points by the intersection of the rays drawn from two instrument stations.

*Procedure* (a) Suppose A and B are two stations and P is an object on the far bank of a river. Now it is required to fix the position of P on the sheet by the intersection of rays, drawn from A and B.

(b) The table is set up at A. It is levelled and centred so that a point *a* on the sheet is just over the station A. The north line is marked on the right-hand top corner. The table is then clamped.

(c) With the alidade touching *a*, the object P and the ranging rod at B are bisected, and rays are drawn through the fiducial edge of the alidade.

(d) The distance AB is measured and plotted to any suitable scale to the [illegible] point b.

(e) The table is shifted and centred over B and levelling, orienting [illegible] alidade is placed along the line ba and orientation is done by back-sighting. [illegible] Here it should be remembered that the centring, levelling and orientation [illegible] be done simultaneously.

(f) With the alidade touching b, the object Q is sighted and a ray is drawn. Suppose this ray intersects the previous ray at a point q. This point q is the plotted position of Q (Fig. 6.11).

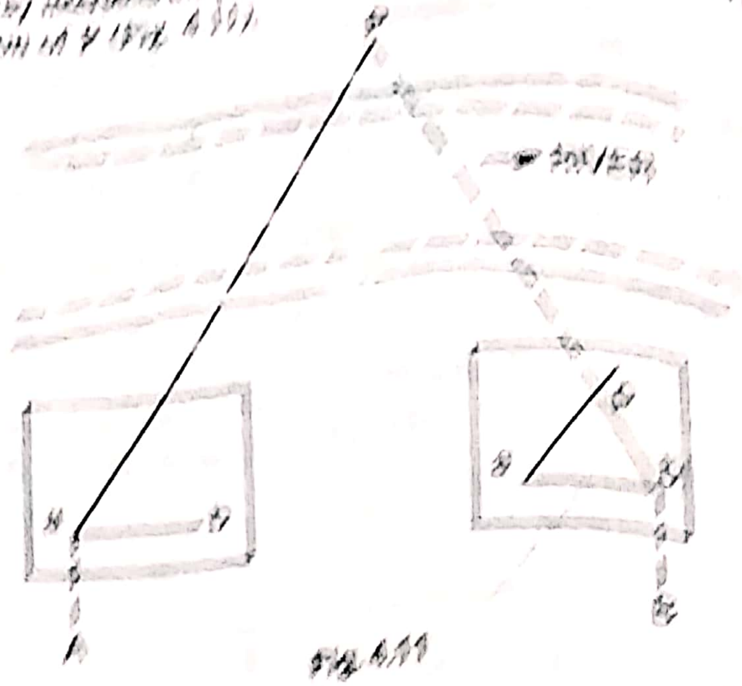


Fig. 6.11

**3. The traversing method** This method is suitable for connecting the [illegible] stations. This is similar to compass traversing or theodolite traversing. The [illegible] levelling and plotting are done simultaneously with the help of the following [illegible] methods.

Suppose A, B, C and D are the traverse stations.

**Procedure** (a) Suppose A, B, C and D [illegible] A. A suitable point a is selected [illegible]

(b) The table is set up at the station A [illegible] sheet in such a way that the whole area may be plotted in the sheet. The table is centred, levelled and clamped. The north line is marked on the right-hand corner of the sheet.

(c) With the alidade touching point a the ranging rod at B is bisected and a ray is drawn. The distance AB is measured and plotted to any suitable scale.

(d) The table is shifted and centred over B. It is then levelled, oriented by back-sighting and clamped.

(e) With the alidade touching point b, the ranging rod at C is bisected and a ray is drawn. The distance BC is measured and plotted to the same scale.

(f) The table is shifted and set up at C and the same procedure is repeated.

(g) In this manner, all stations of the traverse are connected.

(h) At the end, the finishing point may not coincide with the starting point; there may be some closing error. This error is adjusted graphically by [illegible] rule (given in Sec. [illegible]).

(c) [illegible] the table is again set up at [illegible] the surrounding details are located [illegible]

(d) The table is then shifted and set up at all the stations of the traverse and [illegible] the details are located by the radiation and intersection methods (Fig. 4.12).

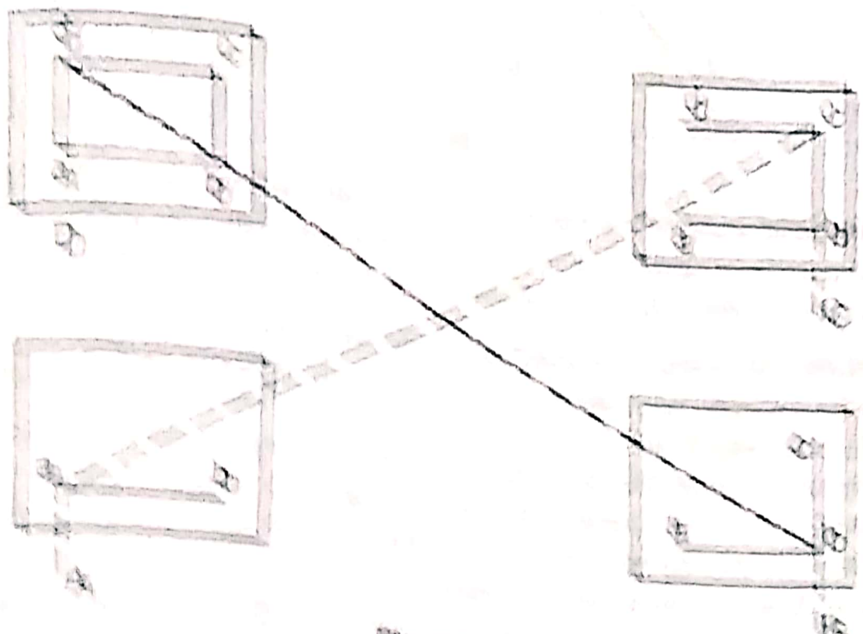


Fig. 4.12

4. *The resection method* This method is suitable for establishing new stations at a place in order to locate missing details.

*Procedure* (a) Suppose it is required to establish a station at position P. Let us select two points A and B on the ground. The distance AB is measured and plotted to any suitable scale. This line AB is known as the "base line".

(b) The table is set up at A. It is levelled, centred and oriented by bisecting the ranging rod at B. The table is then clamped.

(c) With the alidade touching point *a*, the ranging rod at P is bisected and a ray is drawn. Then a point $P_1$ is marked on this ray by estimating with the eye.

(d) The table is shifted and centred in such a way that $P_1$ is just over P. It is then oriented by backsighting the ranging rod at A.

(e) With the alidade touching point *b*, the ranging rod at B is bisected and a ray is drawn. Suppose this ray intersects the previous ray at a point P. This point represents the position of the station P on the sheet. Then the actual position of the station P is marked on the ground by U-fork and plumb bob (Fig. 4.13).

## 4.8 SPECIAL METHODS OF RESECTION

Sometimes, after the completion of plane table traversing, it may be noticed that an important object has not been located due to oversight. If no station pegs are found on the field, some special methods of resection are applied in order to establish a new station for plotting the missing object. The methods are based on: (1) the two-point problem, and (2) the three-point problem.

Fig. 4.13

**1. The two-point problem**  In this problem, two well-defined points whose positions have already been plotted on the plan are selected. Then, by perfectly bisecting these points, a new station is established at the required position.

**Procedure**  (a) Suppose P and Q are two well-defined points whose positions are plotted on plan as $p$ and $q$. It is required to locate a new station at A by perfectly bisecting P and Q.

(b) An auxiliary station B is selected at a suitable position. The table is set up at B, and levelled and oriented by eye estimation. It is then clamped.

(c) With the alidade touching $p$ and $q$, the points P and Q are bisected and rays are drawn. Suppose these rays intersect at $b$.

(d) With the alidade centred on $b$, the ranging rod at A is bisected and a ray is drawn. Then, by eye estimation, a point $a_1$ is marked on this ray.

(e) The table is shifted and centred on A, with $a_1$ just over A. It is levelled and oriented by back sighting. With the alidade touching $p$, the point P is bisected and a ray is drawn. Suppose this ray intersects the line $ba_1$ at point $a_1$, as was assumed previously.

(f) With the alidade centred on $a_1$ the point Q is bisected and a ray is drawn. Suppose this ray intersects the ray $bq$ at a point $q_1$. The triangle $pqq_1$ is known as the triangle of error, and is to be eliminated.

(g) The alidade is placed along the line $pq_1$ and a ranging rod R is fixed at some distance from the table. Then, the alidade is placed along the line $pq$ and the table is turned to bisect R. At this position the table is said to be perfectly oriented.

(h) Finally, with the alidade centred on $p$ and $q$, the points P and Q are bisected and rays are drawn. Suppose these rays intersect at a point $a$. This would represent the exact position of the required station A (Fig. 4.14). Then the station A is marked on the ground.

**2. The three-point problem**  In this problem, three well-defined points are selected whose positions have already been plotted on the map. Then, by perfectly bisecting these three well-defined points, a new station is established at the required position.

No auxiliary station is required in order to solve this problem. The table is directly placed at the required position. The problem may be solved by three methods: (a) the graphical or Bessel's method, (b) the mechanical method, and (c) the trial-and-error method.

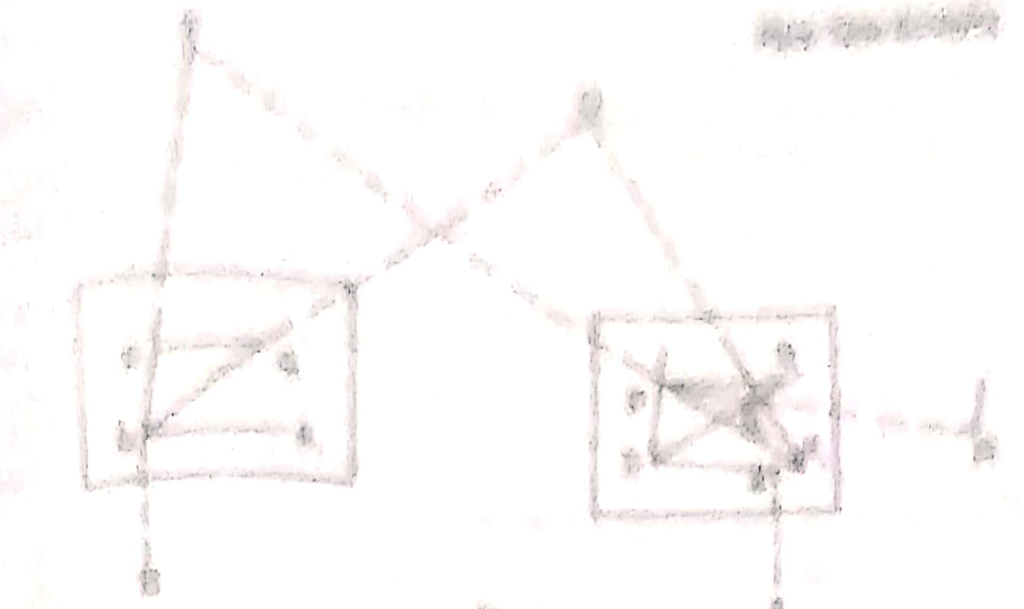


Fig. 4.14

*(a) The Graphical Method* (i) Suppose A, B and C are three well-defined points which have been plotted as a, b, and c. Now it is required to locate a station at P.

(ii) The table is placed at the required station P and levelled. The alidade is placed along the line ca and the point A is bisected. The table is clamped. With the alidade centred on C, the point B is bisected and ray is drawn (Fig. 4.15(a)).

(iii) Again the alidade is placed along the line ac and the point C is bisected and the table is clamped. With the alidade touching a, the point B is bisected and a ray is drawn. Suppose this ray intersects the previous ray at a point *d* (Fig. 4.15(b)).

(iv) The alidade is placed along *db* and the point B is bisected. At this position the table is said to be perfectly oriented. Now the rays Aa, Bb and Cc are drawn. These three rays must meet at a point *p* which is the required point on the map. This point is transferred to the ground by U-fork and plumb bob (Fig. 4.15(c)).

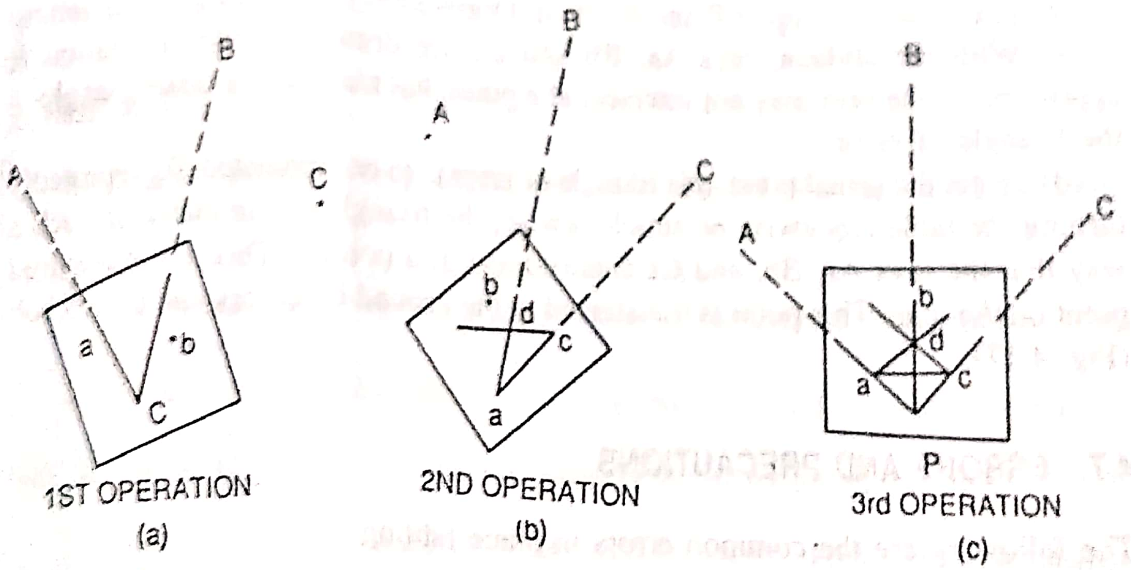



**Fig. 4.15**

*(b) The Mechanical Method* (i) Suppose A, B and C are three well-defined points which have been plotted on the map as a, b, and c. It is required to locate a station at P.

3. Marking the stations. The selected stations are marked on the ground by wooden pegs. [illegible]

4. [illegible] A detailed description is given [illegible]

## [illegible] ADVANTAGES AND DISADVANTAGES OF PLANE TABLING

### 1. Advantages

1. It is the most rapid method of surveying.
2. There is no need for a field book as plotting is done along with the field work. So the problem of mistakes in booking field notes does not arise.
3. Plotted work can be compared with actual object regardless of whether [illegible]
4. There is no possibility of overlooking any important object.
5. There is no possibility of overlooking any measurement as plotting is done in the field.
6. Irregular objects may be represented accurately.
7. It is suitable in magnetic areas.
8. The map can be prepared easily, and does not require any great skill.
9. Errors in measurement and plotting can be detected by check lines.
10. Inaccessible points can be easily located by intersection.

### 2. Disadvantages

1. The plane table is not suitable for accurate work as the fitting arrangement is not perfect.
2. Plane table surveying is not suitable in wet climate, in the rainy season, on foggy mornings and in windy weather.
3. The number of accessories required in such survey is large, and they are likely to be lost.
4. The instrument is very heavy and difficult to carry.
5. The map cannot be replotted to a different scale as there is no field book.

# 5

# Levelling

## 5.1 OBJECT AND USE OF LEVELLING

***Object*** The aim of levelling is to determine the relative heights of different objects on or below the surface of the earth and to determine the undulation of the ground surface.

***Uses*** Levelling is done for the following purposes:

1. To prepare a contour map for fixing sites for reservoirs, dams, barrages, etc., and to fix the alignment of roads, railways, irrigation canals, and so on.
2. To determine the altitudes of different important points on a hill or to know the reduced levels of different points on or below the surface of the earth.
3. To prepare a longitudinal section and cross-sections of a project (roads, railways, irrigation canals, etc) in order to determine the volume of earth work.
4. To prepare a layout map for water supply, sanitary or drainage schemes.

## 5.2 DEFINITIONS

***1. Levelling*** The art of determining the relative heights of different points on or below the surface of the earth is known as levelling. Thus, levelling deals with measurements in the vertical plane.

***2. Level surface*** Any surface parallel to the mean spheroidal surface of the earth is said to be a level surface. Such a surface is obviously curved. The water surface of a still lake is also considered to be a level surface.

***3. Level line*** Any line lying on a level surface is called a level line. This line is normal to the plumb line (direction of gravity) at all points (Fig. 5.1).

***4. Horizontal plane*** Any plane tangential to the level surface at any point is known as the horizontal plane. It is perpendicular to the plumb line which indicates the direction of gravity.

***5. Horizontal line*** Any line lying on the horizontal plane is said to be a horizontal line. It is a straight line tangential to the level line (Fig. 5.1).

*(c) Arbitrary Bench-marks* When the RLs of some fixed points are assumed, they are termed arbitrary bench marks. These are adopted in small survey operations when only the undulation of the ground surface is required to be determined.

*(d) Temporary Bench-marks* When the bench-marks are established temporarily at the end of a day's work, they are said to be temporary bench-marks. They are generally made on the root of a tree, the parapet of a nearby culvert, a furlong post, or on a similar place.

***14. Backsight reading (BS)*** This is the first staff reading taken in any set up of the instrument after the levelling has been perfectly done. This reading is always taken on a point of known RL, i.e. on a bench-mark or change point (Fig. 5.4).

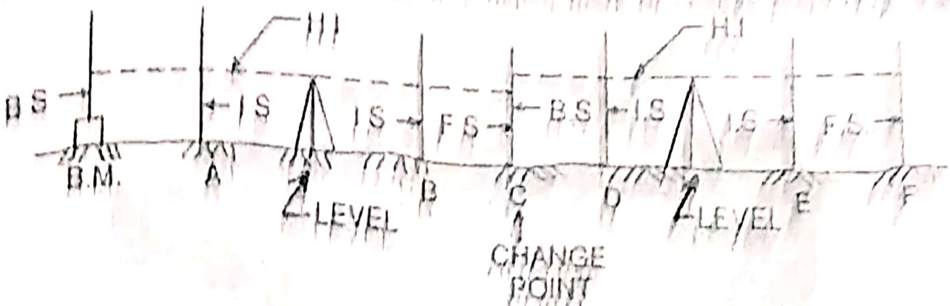



**Fig. 5.4**

***15. Foresight reading (FS)*** It is the last staff reading in any set up of the instrument, and indicates the shifting of the latter (Fig. 5.4).

***16. Intermediate sight reading (IS)*** It is any other staff reading between the BS and FS in the same set up of the instrument (Fig. 5.4).

***17. Change point (CP)*** This point indicates the shifting of the instrument. At this point, an FS is taken from one setting and a BS from the next setting (Fig. 5.4).

***18. Height of instrument (HI)*** When the levelling instrument is properly levelled, the RL of the line of collimation is known as the height of the instrument. This is obtained by adding the BS reading to the RL of the BM or CP on which the staff reading was taken.

***19. Focussing*** The operation of setting the eye-piece and the object glass a proper distance apart for clear vision of the object is known as focussing. This is done by turning the focussing screw clockwise or anticlockwise.

The function of the object glass is to bring the object into focus on the diaphragm, and that of the eye-piece is to magnify the cross-hairs and object.

Focussing is done in two steps as follows.

(a) Focussing the eye-piece: A sheet of white paper is held in front of the telescope and the eye-piece is turned clockwise or anticlockwise slowly until the cross-hairs appear distinct and clear.
(b) Focussing the object glass: The telescope is directed to the object and the focussing screw is turned clockwise or anticlockwise until the image is clear and sharp.

***20. Parallax*** The apparent movement of the image relative to the cross-hairs is

[illegible]

## [illegible] TEMPORARY ADJUSTMENT OF LEVEL

The adjustments made at every set up of the level before the staff readings [illegible] which are known as temporary adjustments. The following are the different steps to be followed in temporary adjustments:

**1. Selection of suitable position** A suitable position is selected for setting up the level. From this position, it should be possible to take the maximum number of [illegible]. The ground should be fairly level and the observations [illegible] without any difficulty.

**2. Fixing level with tripod stand** The tripod stand is placed at the required position with its legs well apart, and pressed firmly into the ground.

The level is fixed on the top of the tripod stand according to the fixing arrangement provided for the particular level. It should be remembered that the level is [illegible] to set up in any position [illegible].

**3. Approximate levelling by legs of tripod stand** The foot screws are brought to the centre of their run. Two legs of the tripod stand are firmly fixed into the ground. Then the third leg is moved to the left or right, in or out until the bubble is approximately at the centre of its run.

**4. Perfect levelling by foot screws** As the longitudinal bubble is on the top of the telescope, the latter is placed parallel to any pair of foot screws (i.e. first position) and the bubble is brought to the centre by turning the foot screws equally, either both inwards or both outwards. The telescope is then turned through 90° (to second position) and brought over the third foot screw, and the bubble is brought to the centre by turning this foot screw clockwise or anticlockwise (Fig. 3.[illegible]). The telescope is again brought to its original position (the first position) and the bubble is brought to the centre. The process is repeated several times until the bubble remains in the central position in the first as well as the second position. Then the telescope is turned through 180°. If the bubble still remains in the central position, the temporary adjustment is perfect and so is the permanent adjustment. But if the bubble is deflected from its central position, the permanent adjustment is not perfect and needs to be modified. (Permanent adjustment is described in Sec. 3.13).

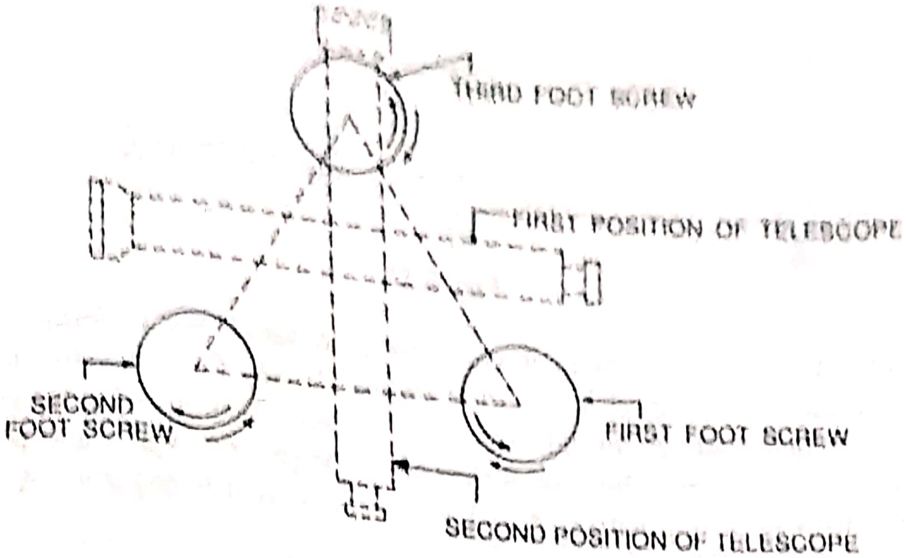



Fig. 5.9

***5. Focussing the eye-piece*** A piece of white paper is held in front of the object glass and the eye-piece is moved in or out by turning it clockwise or anticlockwise until the cross-hairs can be seen clearly.

***6. Focussing the object glass*** The telescope is directed towards the levelling staff. Looking through the eye-piece, the focussing screw is turned clockwise or anticlockwise until the graduation of the staff is distinctly visible and the parallax is eliminated. To eliminate the parallax, the eye is moved up and down to verify whether the graduation of the staff remains fixed relative to the cross-hairs.

***7. Taking the staff readings*** Finally, the levelling of the instrument is verified by turning the telescope in any direction. When the bubbles (the longitudinal bubble and cross bubbles) remain in the central position for any direction of the telescope, the staff readings are taken.

## 5.5 TYPES OF LEVELLING OPERATION

***1. Simple levelling*** When the difference of level between two points is determined by setting the levelling instrument midway between the points, the process is called simple levelling.

Suppose A and B are two points whose difference of level is to be determined. The level is set up at O, exactly midway between A and B. After proper temporary adjustment, the staff readings on A and B are taken. The difference of these readings gives the difference of level between A and B (Fig. 5.10).

***2. Differential levelling*** Differential levelling is adopted when: (i) the points are a great distance apart, (ii) the difference of elevation between the points is large, (iii) there are obstacles between the points.

This method is also known as compound levelling or continuous levelling. In this method, the level is set up at several suitable positions and staff readings are taken at all of these.

Arithmetical check:

$$\Sigma BS - \Sigma FS = [illegible] - [illegible] = + 1.640$$
$$\Sigma Rise - \Sigma fall = [illegible] - [illegible] = + 1.640$$
$$Last\ RL - 1st\ RL = 99.725 - 98.085 = + 1.640$$

## [illegible] PROBLEMS ON REDUCTION OF LEVELS

*Problem 1* The following consecutive readings were taken with a levelling instrument at intervals of 20 m.

2.375, 1.730, 0.615, 3.450, 2.835, 2.070, 1.835, 0.985, 0.435, 1.630, 2.255 and 3.630 m.

The instrument was shifted after the fourth and eighth readings. The last reading was taken on a BM of RL 110.200 m. Find the RLs of all the points.

*Solution*

| Station point | Chainage | BS | IS | FS | Rise (+) | Fall (−) | RL | Remark |
|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 2.375 | | | | | | |
| 2 | 20 | | 1.730 | | | | 112.620 | |
| 3 | 40 | | 0.615 | | 0.645 | | 113.265 | |
| 4 | 60 | 2.835 | | | 1.115 | | 114.380 | |
| | | | | 3.450 | | 2.835 | 111.545 | Change point |
| 5 | 80 | | 2.070 | | | | | |
| 6 | 100 | | 1.835 | | 0.765 | | 112.310 | |
| 7 | 120 | 0.435 | | | 0.235 | | 112.545 | |
| | | | | 0.985 | 0.850 | | 113.395 | Change point |
| 8 | 140 | | 1.630 | | | | | |
| 9 | 160 | | 2.255 | | | 1.195 | 112.200 | |
| 10 | 180 | | | | | 0.625 | 111.575 | |
| | | | | 3.630 | | 1.375 | 110.200 | On BM |
| Total = | | 5.645 | | 8.065 | 3.610 | 6.030 | | |

*Procedure:*

1. First calculate the rise and fall. Then find:

$$\Sigma BS - \Sigma FS = 5.645 - 8.065 = - 2.420$$
$$\Sigma Rise - \Sigma fall = 3.610 - 6.030 = - 2.420$$

2. Then, Last RL − 1st RL = − 2.420

or 1st RL = 110.200 + 2.420 = 112.620

Substitute this value for the RL of the first point and calculate the other RLs in the usual way.

*Problem 2* The following successive readings were taken with a dumpy level along a chain line at common intervals of 20 m. The first reading was taken on

**Example** [illegible]

[illegible]

**Solution**

[illegible]

| Station | B.S. | I.S. | F.S. | Rise | Fall | R.L. | Remarks |
|---|---|---|---|---|---|---|---|
| 1 | [illegible] | | | | | [illegible] | [illegible] |
| 2 | | [illegible] | | [illegible] | | [illegible] | |
| 3 | [illegible] | | [illegible] | | [illegible] | [illegible] | [illegible] |
| 4 | | [illegible] | | | [illegible] | [illegible] | |
| 5 | [illegible] | | [illegible] | [illegible] | | [illegible] | [illegible] |
| 6 | [illegible] | | [illegible] | | [illegible] | [illegible] | [illegible] |
| 7 | | | [illegible] | | [illegible] | [illegible] | |
| Check | [illegible] | | [illegible] | [illegible] | [illegible] | [illegible] | |
| | | Fall | [illegible] | Fall | [illegible] | [illegible] | Checked |

# 9 Theodolite Traversing

## 9.1 INTRODUCTION

The theodolite is an intricate instrument used mainly for accurate measurement of horizontal and vertical angles up to 10" or 20", depending upon the least count of the instrument. Because of its various uses, the theodolite is sometimes known as a universal instrument. The following are the different purposes for which the theodolite can be used:

1. Measuring horizontal angles
2. Measuring vertical angles
3. Measuring deflection angles
4. Measuring magnetic bearings
5. Measuring the horizontal distance between two points
6. Finding the vertical height of an object
7. Finding the difference of elevation between various points
8. Ranging a line

Theodolites may be of two types—(i) transit theodolite, and (ii) non-transit. In the transit theodolite, the telescope can be revolved through a complete revolution about its horizontal axis in a vertical plane.

In the non-transit theodolite, the telescope cannot be revolved through a complete revolution in the vertical plane. But it can be revolved to a certain extent in the vertical plane, in order to measure the angle of elevation or depression.

Theodolites may also be classified as: (i) vernier theodolites—when fitted with a vernier scale, and (ii) micrometer theodolites—when fitted with a micrometer.

The size of the theodolite is defined according to the diameter of the main horizontal graduated circle. For example, in a "10 cm theodolite", the diameter of the main graduated circle is 10 cm. In engineering survey, 8 cm to 12 cm theodolites are generally used.

## 9.2 DEFINITIONS

**1. Centring** The setting of a theodolite exactly over a station mark by means of a plumb-bob is known as centring. The plumb-bob is suspended from a hook fixed below the vertical axis.

7. The mean of the two observations will give the correct magnetic bearing of the line.

## 9.12 MEASURING HORIZONTAL DISTANCE BY STADIA METHOD

Consider Fig. 9.13. Suppose the horizontal distance between points A and B is to be determined by a theodolite. The following procedure is adopted.

1. The theodolite is set up at A. It is then centred and levelled properly. The vertical verniers (generally C and D) are set at the "zero" of the vertical circle by turning the clamp screw and the tangent screw of the vertical circle (since the vertical circle is fixed to the telescope).

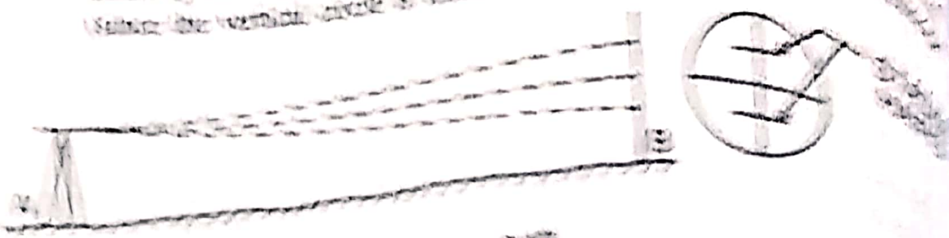


Fig. 9.13

2. The plate bubble is brought to the centre by turning the foot screws, and the altitude bubble is centred by means of the clip screw (which is fixed below the altitude bubble). The line of collimation is made exactly horizontal in this manner.
3. The levelling staff is held at B, and the telescope is directed towards that point. By focussing the staff, the readings of the upper and lower stadia hairs are noted. The difference between the two readings is calculated.
4. The required distance AB is obtained by multiplying this difference by the stadia constant, which is generally 100.

**Example** Suppose the readings of the upper and lower stadia are 3.255 and 1.365 respectively. Then

$$\text{Distance } AB = (3.255 - 1.365) \times 100 = 189 \text{ m}$$

## 9.13 RANGING AND EXTENDING A LINE

### A. Ranging a Line

Ranging is the process of establishing intermediate points on a straight line between the terminal points. Let AB (Fig. 9.14) be the straight line on which intermediate points are to be fixed by theodolite. The procedure is as follows:

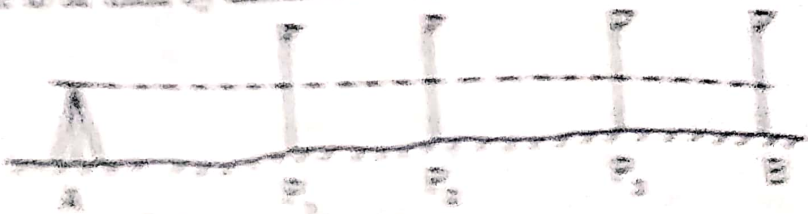



Fig. 9.14

C. Natural Errors

1. High temperature causes error due to irregular refraction.
2. High winds cause vibration in the instrument, and this may lead to wrong readings on the verniers.

## 9.17 CLOSING ERROR AND ITS LIMITATION

In a closed traverse the algebraic sum of latitudes must be equal to zero, so should the algebraic sum of departures.

But due to the errors in field measurements of angles and lengths, the finishing point may not coincide with the starting point of a closed traverse.

The distance by which a traverse fails to close is known as closing error or error of closure.

In Fig. 9.20, the traverse $ABCDA_1$ fails to close by a distance $AA_1$. This is the closing error of this traverse.

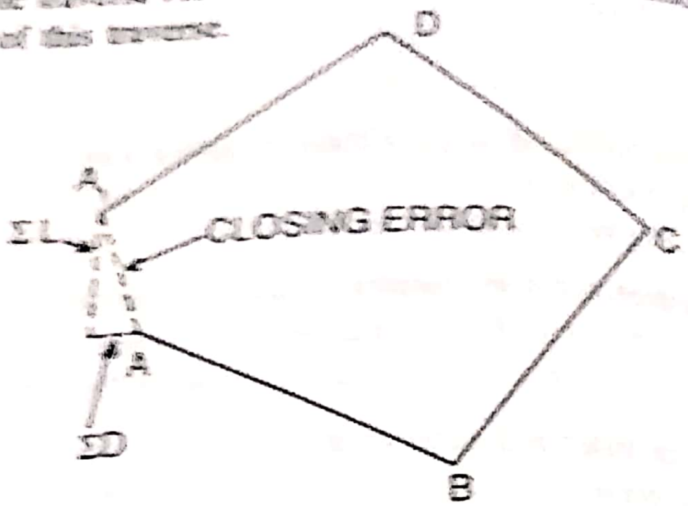



Fig. 9.20

$$\text{Closing error, } AA_1 = \sqrt{(\Sigma L)^2 + (\Sigma D)^2}$$

where $L$ = latitude

and $D$ = departure

$$\text{Relative closing error} = \frac{\text{closing error}}{\text{perimeter of traverse}}$$

$$\text{Permissible angular error} = \text{least count} \times \sqrt{N}$$

where $N$ = number of sides

$$\tan\theta = \frac{\Sigma D}{\Sigma L}$$

where $\theta$ indicates the direction of closing error.

7

# Computation of Area

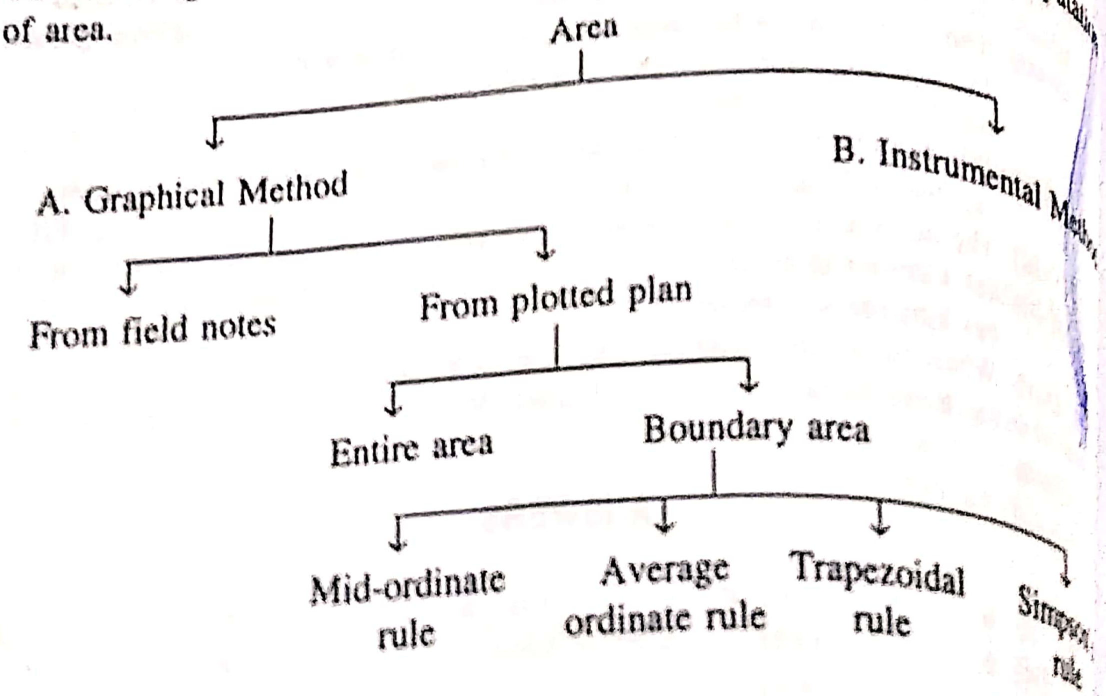
## 7.1 INTRODUCTION

The term 'area' in the context of surveying refers to the area of a tract of land projected upon the horizontal plane, and not to the actual area of the land surface.

Area may be expressed in the following units:

1. Square-metres
2. Hectares (1 hectare = 10,000 $m^2$)
3. Square-feet
4. Acres (1 acre = 4840 sq. yd. = 43.560 sq. ft.)

The following is a hierarchical representation of the various methods of computation of area.




## 7.2 COMPUTATION OF AREA FROM FIELD NOTES

This is done in two steps.

**Step 1** In cross-staff survey, the area of field can be directly calculated from field notes. During survey work the whole area is divided into some geometrical figures, such as triangles, rectangles, squares, and trapeziums, and then the area is calculated as follows:

## 7.4 COMPUTATION OF AREA FROM PLOTTED PLAN

The area may be calculated in the two following ways.

***Case I—Considering the entire area*** The entire area is divided into regions of a convenient shape, and calculated as follows:

*(a) By dividing the area into triangles* The triangles are so drawn as to equalise the irregular boundary line.

Then the bases and altitudes of the triangles are determined according to the scale to which the plan was drawn. After this, the areas of these triangles are calculated (area = 1/2 × base × altitude). The areas are then added to obtain the total area (Fig. 7.6).

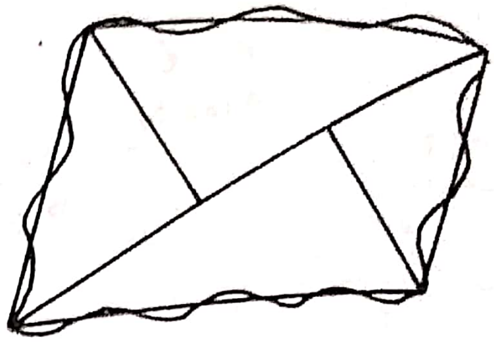


Fig. 7.6

*(b) By dividing the area into squares* In this method, squares of equal size are ruled out on a piece of tracing paper. Each square represents a unit area, which could be 1 cm$^2$ or 1 m$^2$. The tracing paper is placed over the plan and the number of full squares are counted. The total area is then calculated by multiplying the number of squares by the unit area of each square (Fig. 7.7).

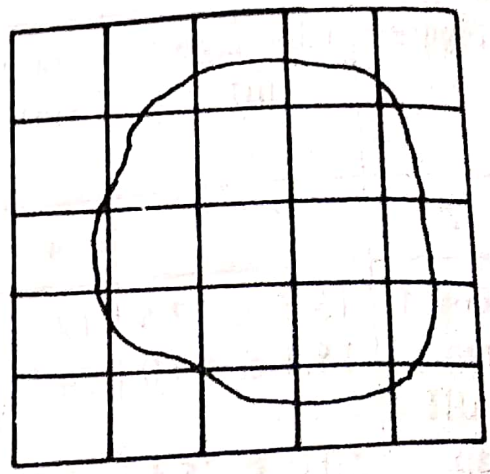


Fig. 7.7

*(c) By drawing parallel lines and converting them to rectangles* In this method, a series of equidistant parallel lines are drawn on a tracing paper (Fig. 7.8). The constant distance represents a metre or centimetre. The tracing paper is placed

[illegible] The sum of the lengths of rectangles is then calculated.

Required area = Σ length of rectangles × constant distance

Fig. 7.8

Case II In this method, a large square or rectangle is formed within the area in the plan. Then ordinates are drawn at regular intervals from the side of the square to the curved boundary. The middle area is calculated in the usual way. The boundary area is calculated according to one of the following rules:

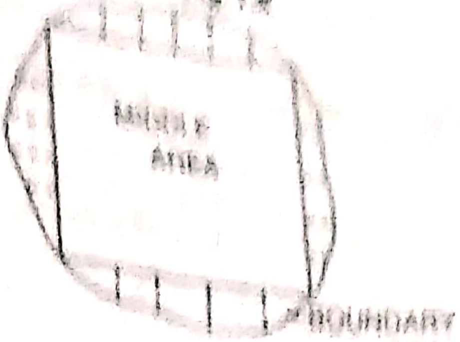



Fig. 7.9

1. The mid-ordinate rule
2. The average ordinate rule
3. The trapezoidal rule
4. Simpson's rule

The various rules are explained in the following sections.

## 7.5 THE MID-ORDINATE RULE

Consider Fig. 7.10

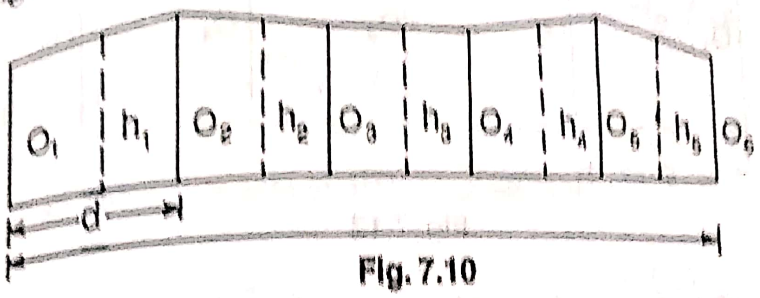



Fig. 7.10

Let $O_1, O_2, O_3, \ldots, O_n$ = ordinates at equal intervals

$l$ = length of base line

$d$ = common distance between ordinates

$h_1, h_2, \ldots h_n$ = mid-ordinates

$$\text{Area of plot} = h_1 \times d + h_2 \times d + \ldots + h_n \times d$$
$$= d(h_1 + h_2 + \ldots + h_n)$$

i.e. Area = common distance × sum of mid-ordinates

## 7.6 THE AVERAGE ORDINATE RULE

Refer to Fig. 7.11

Fig. 7.11

Let $O_1, O_2, \ldots, O_n$ = ordinates or offsets at regular intervals

$l$ = length of base line

$n$ = number of divisions

$n + 1$ = number of ordinates

$$\text{Area} = \frac{O_1 + O_2 + \ldots + O_n}{O_{n+1}} \times l$$

i.e. $$\text{Area} = \frac{\text{sum of ordinates}}{\text{no. of ordinates}} \times \text{length of base line}$$

## 7.7 THE TRAPEZOIDAL RULE

While applying the trapezoidal rule, boundaries between the ends of ordinates are assumed to be straight. Thus the areas enclosed between the base line and the irregular boundary line are considered as trapezoids.

Consider Fig. 7.12.

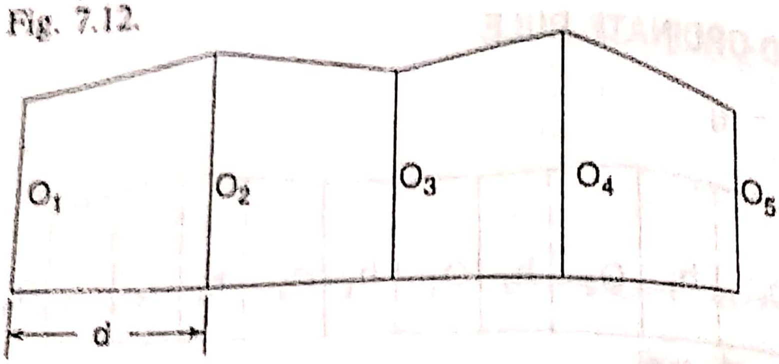



Fig. 7.12

Let $O_1, O_2, \ldots, O_n$ = ordinates at equal intervals

$d$ = common distance

$$\text{1st area} = \frac{O_1 + O_2}{2} \times d$$

$$\text{2nd area} = \frac{O_2 + O_3}{2} \times d$$

$$\text{3rd area} = \frac{O_3 + O_4}{2} \times d$$

## SIMPSON'S RULE

In this rule, the boundaries between the ends of ordinates are assumed to form an arc of a parabola. Hence Simpson's rule is sometimes called the parabolic rule. Refer to Fig. 7.13.

Let

$O_1, O_2, O_3$ = three consecutive ordinates

$d$ = common distance between the ordinates

area $AFeDC$ = area of trapezium $AFDC$ + area of segment $FeDEF$

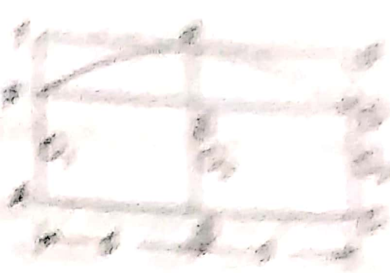


Fig. 7.13

Here,

Area of trapezium $= \frac{O_1 + O_3}{2} \times 2d$

Area of segment $= \frac{2}{3} \times$ area of parallelogram FfdD

$$= \frac{2}{3} \times Ee \times 2d = \frac{2}{3} \times \left\{O_2 - \frac{O_1 + O_3}{2}\right\} \times 2d$$

So, the area between the first two divisions,

$$\Delta_1 = \frac{O_1 + O_3}{2} \times 2d + \frac{2}{3}\left\{O_2 - \frac{O_1 + O_3}{2}\right\} \times 2d$$

$$= \frac{d}{3}(O_1 + 4O_2 + O_3)$$

Similarly, the area between next two divisions,

$\Delta_2 = \frac{d}{3}(O_3 + 4O_4 + O_5)$ and so on

Total area $= \frac{d}{3}(O_1 + 4O_2 + 2O_3 + 4O_4 + \dots + O_n)$

$= \frac{d}{3}\{O_1 + O_n + 4(O_2 + O_4 + \dots) + 2(O_3 + O_5 + \dots)\}$

$= \frac{\text{common distance}}{3}$ {(1st ordinate + last ordinate)

+ 4 (sum of even ordinates)

+ 2 (sum of remaining odd ordinates)}

Thus, the rule may be stated as follows.

To the sum of the first and the last ordinate, four times the sum of even ordinates and twice the sum of the remaining odd ordinates are added. This sum is multiplied by the common distance. One-third of this product is the required area.

*Limitation* This rule is applicable only when the number divisions is even, i.e. the number of ordinates is odd.

The trapezoidal rule and Simpson's rule may be compared in the following manner:

| *Trapezoidal rule* | *Simpson's rule* |
|---|---|
| 1. The boundary between the ordinates is considered to be straight. | 1. The boundary between the ordinates is considered to be an arc of a parabola. |
| 2. There is no limitation. It can be applied for any number of ordinates. | 2. To apply this rule, the number of ordinates must be odd. That is, the number of divisions must be even. |
| 3. It gives an approximate result. | 3. It gives a more accurate result. |

*Note* Sometimes one, or both, of the end ordinates may be zero. However, they must be taken into account while applying these rules.

## 7.9 WORKED-OUT PROBLEMS

*Problem 1* The following offsets were taken from a chain line to an irregular boundary line at an interval of 10 m:

0, 2.50, 3.50, 5.00, 4.60, 3.20, 0 m

compute the area between the chain line, the irregular boundary line and the end offsets by:

(a) The mid-ordinate rule

(a) By mid-ordinate rule: The mid-ordinates are

$$h_1 = \frac{0 + 2.50}{2} = 1.25 \text{ m}$$

$$h_2 = \frac{2.50 + 3.50}{2} = 3.00 \text{ m}$$

$$h_3 = \frac{3.50 + 5.00}{2} = 4.25 \text{ m}$$

$$h_4 = \frac{5.00 + 4.60}{2} = 4.80 \text{ m}$$

$$h_5 = \frac{4.60 + 3.20}{2} = 3.90 \text{ m}$$

$$h_6 = \frac{3.20 + 0}{2} = 1.60 \text{ m}$$

$$\text{Required area} = 10\,(1.25 + 3.00 + 4.25 + 4.80 + 3.90 + 1.60)$$
$$= 10 \times 18.80 = 188 \text{ m}^2$$

(b) By average-ordinate rule:

Here $d = 10$ m and $n = 6$ (no. of divs)

Base length $= 10 \times 6 = 60$ m

Number of ordinates $= 7$

$$\text{Required area} = 60 \times \left\{\frac{0 + 2.50 + 3.50 + 5.00 + 4.60 + 3.20 + 0}{7}\right\}$$
$$= 60 \times \frac{18.80}{7} = 161.14 \text{ m}^2$$

(c) By trapezoidal rule:

Here $d = 10$

$$\text{Required area} = \frac{10}{2}\,\{0 + 0 + 2(2.50 + 3.50 + 5.00 + 4.60 + 3.20)\}$$
$$= 5 \times 37.60 = 188 \text{ m}^2$$

(d) *By Simpson's rule:*

$d = 10$

$$\text{Required area} = \frac{10}{3}\{0 + 0 + 4(2.50 + 5.00 + 3.20) + 2(3.5\ldots + \ldots$$

$$= \frac{10}{3}\{42.80 + 16.20\} = \frac{10}{3} \times 59.00$$

$$= \frac{10}{3} \times 59.00 = 196.66 \text{ m}^2$$

$$A_2 = \frac{15}{2}(4.40 + 3.25) = 57.38 \text{ m}^2$$

$$\text{Total area} = A_1 + A_2 = 756.00 + 57.38 = 813.38 \text{ m}^2$$

*Problem 3* The following offsets are taken from a survey line to a curved boundary line:

| Distance (m) | 0 | 5 | 10 | 15 | 20 | 30 | 40 | 60 | 80 |
|---|---|---|---|---|---|---|---|---|---|
| Offset (m) | 2.50 | 3.80 | 4.60 | 5.20 | 6.10 | 4.70 | 5.80 | 3.90 | 2.20 |

Find the area between the survey line, the curved boundary line, and the first and the last offsets by:

(i) The trapezoidal rule, and (ii) Simpson's rule.

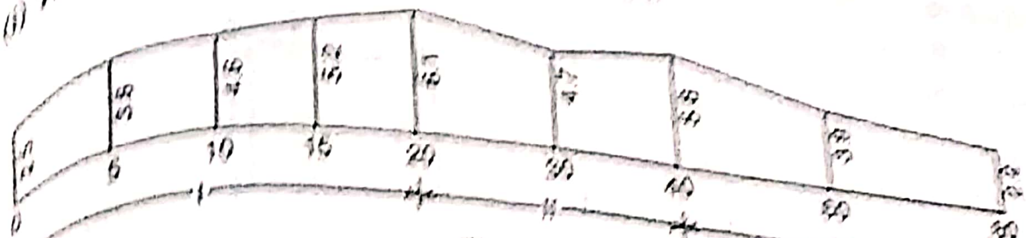


Fig. P.7.3

*Solution* Here, the intervals between the offsets are not regular throughout the length.

So, the section is divided into three compartments.

Let
$A_I$ = area of 1st section
$A_{II}$ = area of 2nd section
$A_{III}$ = area of 3rd section

Here,
$d_1$ = 5 m
$d_2$ = 10 m
$d_3$ = 20 m

*(a) By Trapezoidal rule:*

$$A_I = \frac{5}{2}\{2.50 + 6.10 + 2(3.80 + 4.60 + 5.20)\} = 89.50 \text{ m}^2$$

$$A_{II} = \frac{10}{2}\{6.10 + 5.80 + 2(4.70)\} = 106.50 \text{ m}^2$$

$$A_{III} = \frac{20}{2}\{5.80 + 2.20 + 2(3.90)\} = 158.00 \text{ m}^2$$

$$\text{Total area} = 89.50 + 106.50 + 158.00 = 354.00 \text{ m}^2$$

*(b) By Simpson's rule:*

$$A_I = \frac{5}{3}\{2.50 + 6.10 + 4(3.80 + 5.20) + 2(4.60)\} = 89.66 \text{ m}^2$$

$$A_{II} = \frac{10}{3}\{6.10 + 5.80 + 4(4.70)\} = 102.33 \text{ m}^2$$

$$A_{III} = \frac{20}{3}(5.80 + 2.20 + 4(3.90)) = 157.33 \text{ sq m}$$

$$\text{Total area} = 89.66 + 102.33 + 157.33 = 349.32 \text{ m}^2.$$

# 8

# Computation of Volume

## 8.1 INTRODUCTION

[illegible] computation of the volume of earth work, the sectional areas of the [illegible] which are taken transverse to the longitudinal section during profile levelling [illegible] calculated. Again, the cross-sections may be of different types, namely [illegible] two-level, (iii) three-level, (iv) side-hill two-level, and (v) multi-level. [illegible] methods of calculating the areas of such sections are discussed in Sec. 8.2. [illegible] calculation of cross-sectional areas, the volume of earth work is calculated [illegible] the trapezoidal (or average end area) rule, and (ii) the prismoidal rule.

[illegible]

1. The prismoidal rule gives the correct volume directly.
2. The trapezoidal rule does not give the correct volume. Prismoidal correction should be applied for this purpose. This correction is always subtractive.
3. Cutting is denoted by a positive sign and filling by a negative sign.

## 8.2 FORMULAE FOR CALCULATION OF CROSS-SECTIONAL AREA

### a. Level Section

When the ground is level along the transverse direction:

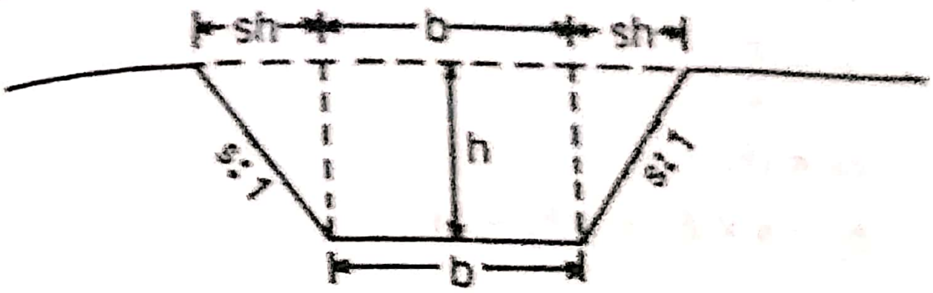



Fig. 8.1

$$\text{Cross-sectional area} = \frac{b + b + 2sh}{2} \times h$$

$$= (b + sh)h \quad (1)$$

## 8.3 FORMULA FOR CALCULATION OF VOLUME

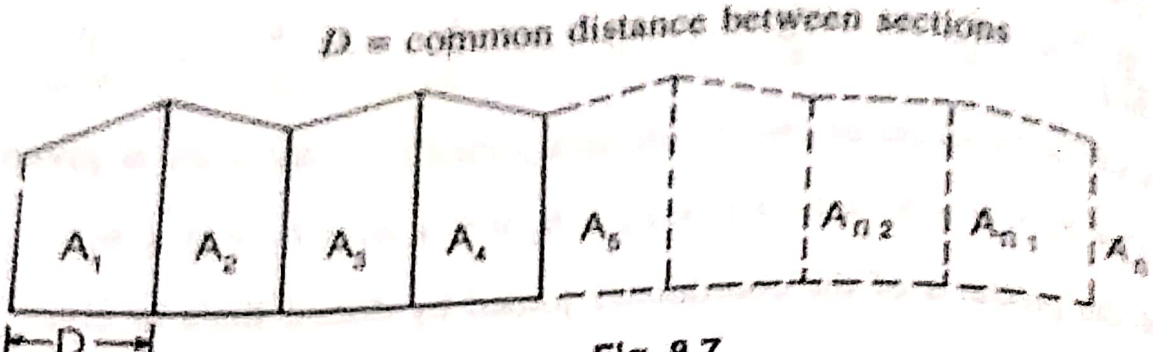



Fig. 8.7

**A. Trapezoidal Rule (Average End Area Rule)**

Volume (cutting or filling), $V = \frac{D}{2}\{A_1 + A_n + 2(A_2 + A_3 + \ldots + A_{n-1})\}$

i.e. $\quad \text{volume} = \frac{\text{common distance}}{2}$ {area of 1st section + area of last section + 2 (sum of area of other sections)}

**B. Prismoidal Formula**

Volume (cutting or filling), $V = \frac{D}{3}\{A_1 + A_n + 4(A_2 + A_4 + A_{n-1}) + 2(A_3 + A_5 + \ldots + A_{n-2})\}$

i.e., $\quad V = \frac{\text{common distance}}{3}$ {Area of 1st section + area of last section + 4 (sum of areas of even sections) + 2 (sum of areas of odd sections)}

## 3.4 WORKED-OUT PROBLEMS

*Problem 1* An embankment of width 10 m and side slopes $1\frac{1}{2} : 1$ is required to be made on a ground which is level in a direction transverse to the centre line. The central heights at 40 m intervals are as follows:

0.90, 1.25, 2.15, 2.50, 1.85, 1.35, and 0.85

Calculate the volume of earth work according to (i) the trapezoidal formula, and (ii) the prismoidal formula.

**Solution** The cross-sectional areas are calculated by Eq. (1):

Area,
$$\Delta = (b + Sh) \times h$$
$$\Delta_1 = (10 + 1.5 \times 0.90) \times 0.90 = 10.22\ m^2$$
$$\Delta_2 = (10 + 1.5 \times 1.25) \times 1.25 = 14.84\ m^2$$
$$\Delta_3 = (10 + 1.5 \times 2.15) \times 2.15 = 28.43\ m^2$$
$$\Delta_4 = (10 + 1.5 \times 2.50) \times 2.50 = 34.38\ m^2$$
$$\Delta_5 = (10 + 1.5 \times 1.85) \times 1.85 = 23.63\ m^2$$
$$\Delta_6 = (10 + 1.5 \times 1.35) \times 1.35 = 16.23\ m^2$$
$$\Delta_7 = (10 + 1.5 \times 0.85) \times 0.85 = 9.58\ m^2$$

(a) Volume according to trapezoidal formula:

$$V = \frac{40}{2}\{10.22 + 9.58 + 2(14.84 + 28.43 + 34.38 + 23.63 + 16.23)\}$$
$$= 20\{19.80 + 235.02\} = 5{,}096.4\ m^3$$

(b) Volume calculated in prismoidal formula:

$$V = \frac{40}{3}\{10.22 + 9.58 + 4(14.84 + 34.38 + 16.23) + 2(28.43 + 23.63)\}$$
$$= \frac{40}{3}(19.80 + 261.80 + 104.12) = 5{,}142.9\ m^3$$