

S.D-II  
Design of footings, (L.S.M)

Foundation is that part of the building whose function is to distribute the loads of the structure to the soil supporting the structure.

Any structure is generally considered to have two main portions.

(i) The superstructure and (ii) substructure.

The substructure transmits the loads of superstructure to the supporting soil and is generally termed as the foundation.

Footing: is that portion of the foundation which ultimately delivers the load to the soil, and is thus in contact with it. Load of the superstructure is transmitted to the foundation through either columns or walls.

Foundations may be classified as

(1) Shallow foundation.

(2) Deep foundation.

Shallow foundation: A foundation is shallow

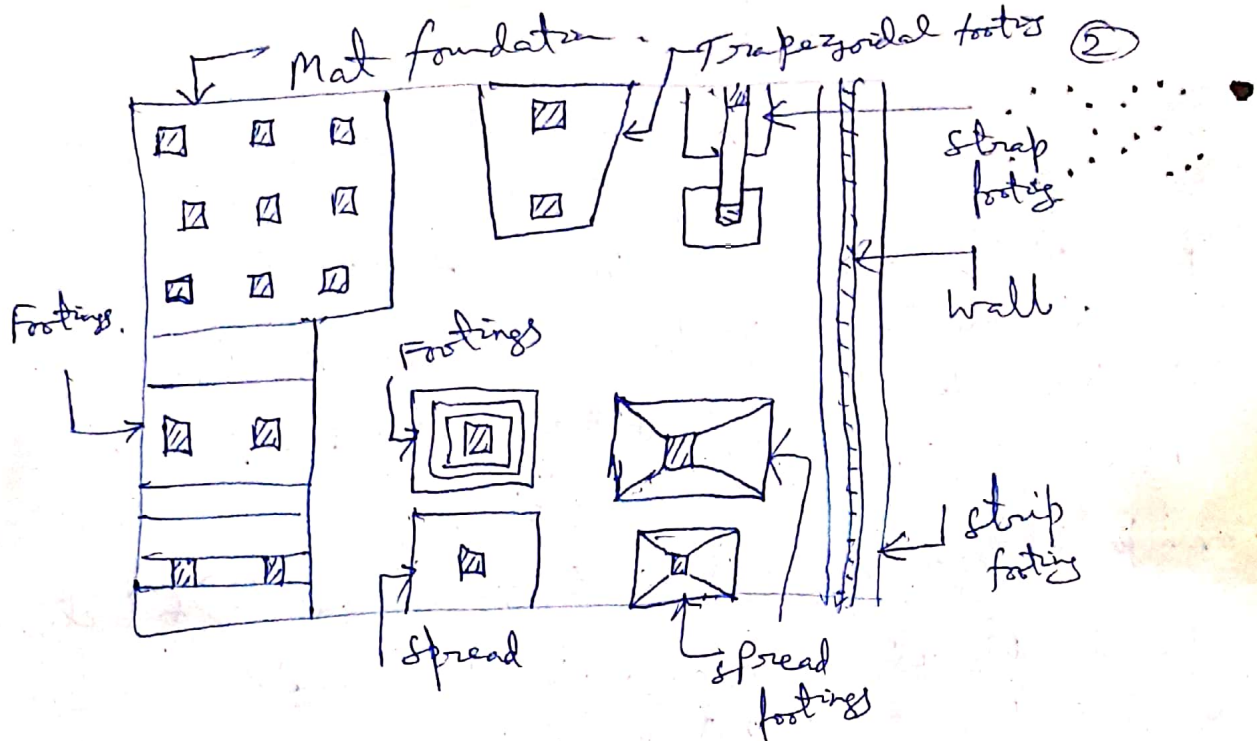
if its depth is equal to or less than

its width. Shallow foundations are of

~~Deep foundation~~ following types.

(1) Spread footing, (2) Strap footing,

(3) Combined footing and (4) mat or Raft footing.



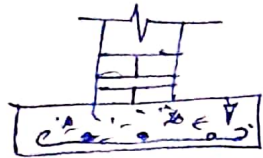
Deep foundation : the depth is greater than the width.

Various forms of deep foundations are

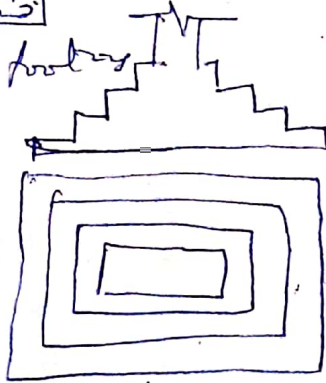
- ① deep strip,
- ② Rectangular or square foundations.
- ③ Pier foundations,
- ④ Pile "
- ⑤ Well "

Safe bearing Capacity : The max intensity of loading that the soil will safely carry without risk of shear failure irrespective of any settlement that may occur.

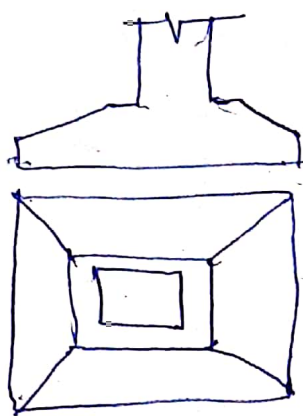
Ultimate Bearing Capacity : The intensity of loading at the base of a foundation which will cause shear failure of the soil support.



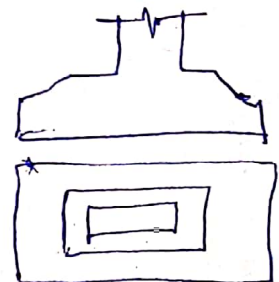
Wall footing



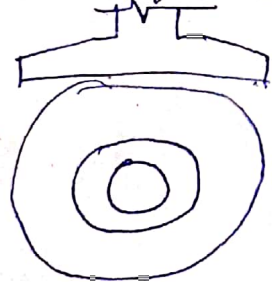
stepped footing.



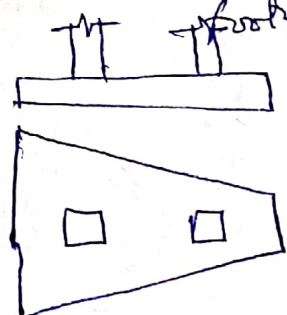
square footing for square column.



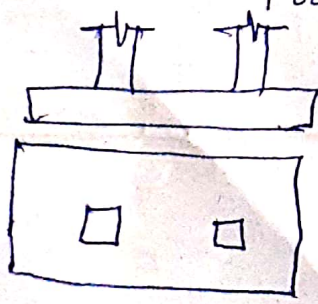
Rectangular footing.



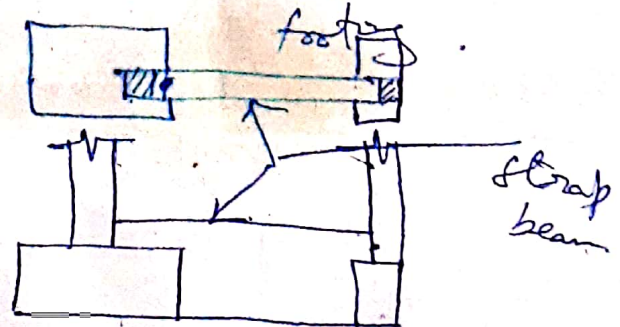
Circular footing



Trapezoidal footing



Rectangular footing.



strap footing.

strap beam

Zan

## S.D-II      Design of footing

Q3/ An R.C.C column of size  $350\text{ mm} \times 350\text{ mm}$  carries a characteristic load of  $750\text{ kN}$ . The S.B.C of soil is  $200\text{ kN/m}^2$ . Design isolated square footing. Use M20 grade of concrete and Fe 415 reinforcement for both column & footing.

Column size =  $350 \times 350\text{ mm}$

$$\text{Load} = 750\text{ kN}$$

$$\text{S.B.C of soil} = 200\text{ kN/m}^2$$

M20 & Fe 415 grade of concrete and steel are used.

$$f_{ck} = 20\text{ N/mm}^2$$

$$f_y = 415\text{ N/mm}^2$$

Total load = characteristic load + foundation load

$$= 750 + 10\% \text{ of the column load}$$

$$= 750 + \left(\frac{10}{100} \times 750\right) = 825\text{ kN}$$

$$\text{Ultimate load} = 1.5 \times 825 = 1237.5\text{ kN}$$

$$A_{req} = \frac{\text{Ultimate load}}{\text{S.B.C}} = \frac{1237.5\text{ kN}}{200}$$

$$= 6.18$$

$$B = 2.5\text{ m}$$

$$L = 2.5 \text{ for square footing}$$

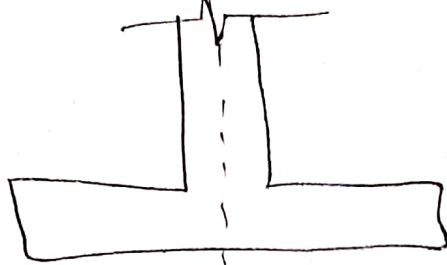
$$A_{provided} = 6.25\text{ m}^2$$

$$\text{upward pressure (q)} = \frac{\text{Wt of Coln}}{A_{provided}}$$

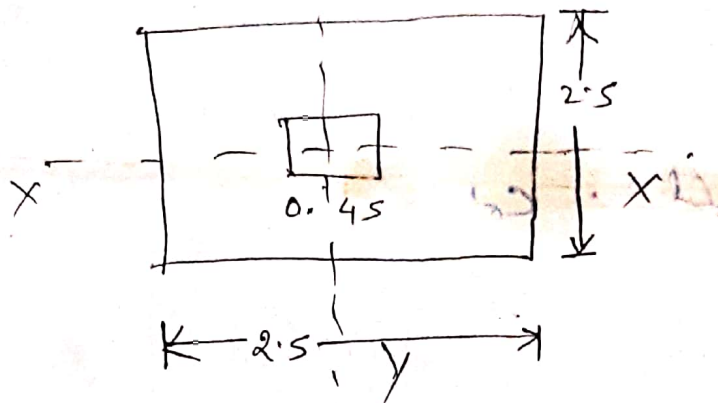
$$= \frac{750 \times 1.5}{6.25} = 180 < \text{S.B.C.}$$

$$\uparrow \text{ i.e. } 200\text{ kN/m}^2 \text{ (S.B.C.)} \quad \text{P.I.D}$$

B.M. Calculations.



slenderness ratio



$$M_{u_{xx}} = M_{u_{yy}}$$

$$M_{u_{xx}} = 2 \times B \times \frac{(B-b)^2}{8} = 180 \times 2.5 \times \frac{(2.5-0.35)^2}{8}$$

Column size ↑

$$M_{u_{xx}} = 260.01 \text{ kN/m}$$

Depth Calculation:

$$M_{u_{lim}} = 0.36 f_{ck} \frac{x_{u_{max}}}{d} \left( 1 - 0.42 \frac{x_{u_{max}}}{d} \right) b d^2$$

$$260.01 \times 10^6 = 0.36 \times 20 \times 0.48 \left( 1 - 0.42 \times 0.48 \right) b d^2$$

$$d = 195 \text{ mm} \approx 350 \text{ mm}$$

$$\therefore D = d + 50$$

$$= 350 + 50 = 400 \text{ mm}$$

Steel Calculation.

$$(A_{st})_x = (A_{st})_y$$

$$\frac{M_u}{b d^2} = \frac{260.01 \times 10^6}{2500 \times (350)^2} = 0.85$$

$$P_t = 0.412 \text{ (I.S. Code)}$$

V.I.O

$$(A_{st})_{xx} = 3605 \text{ mm}^2. \quad \text{pt\% } bd = \frac{0.412}{100} \times 2500 \times 350 \quad (3)$$

$$A_{st} (\text{min}) = \frac{0.12}{100} \times 2500 \times 400$$

$\downarrow$                        $\downarrow$   
 B                              D

$$= 1200 \text{ mm}^2$$

$$\text{Allowable bearing stress} = 0.45 f_{ck} \sqrt{\frac{A_1}{A_2}}$$

$$A_1 = L_1 \times B_1$$

$$L_1 = B + 4D$$

$$= 350 + 4 \times 400 = 1950 \text{ mm}$$

$$A_1 = 1950 \times 1950 = 3802500 \text{ mm}^2$$

$$A_2 = 350 \times 350 = 122500 \text{ mm}^2$$

size of column.

$$\sqrt{\frac{A_1}{A_2}} = 5.57 \neq 2. \text{ so it is taken as } 2.$$

$$\therefore \text{Allowable bearing stress} = 0.45 f_{ck} \sqrt{\frac{A_1}{A_2}}$$

$$= 0.45 \times 20 \times 2 = 18 \text{ N/mm}^2$$

$$\text{Col}^n \text{ load} = 1.5 \times \text{Load}$$

$$= 1.5 \times 750 = 1125 \text{ kN}$$

$$\text{stress} = \frac{\text{Col}^n \text{ load}}{\text{Area}} = \frac{1125 \times 10^3}{350 \times 350} = 9.18 < 18$$

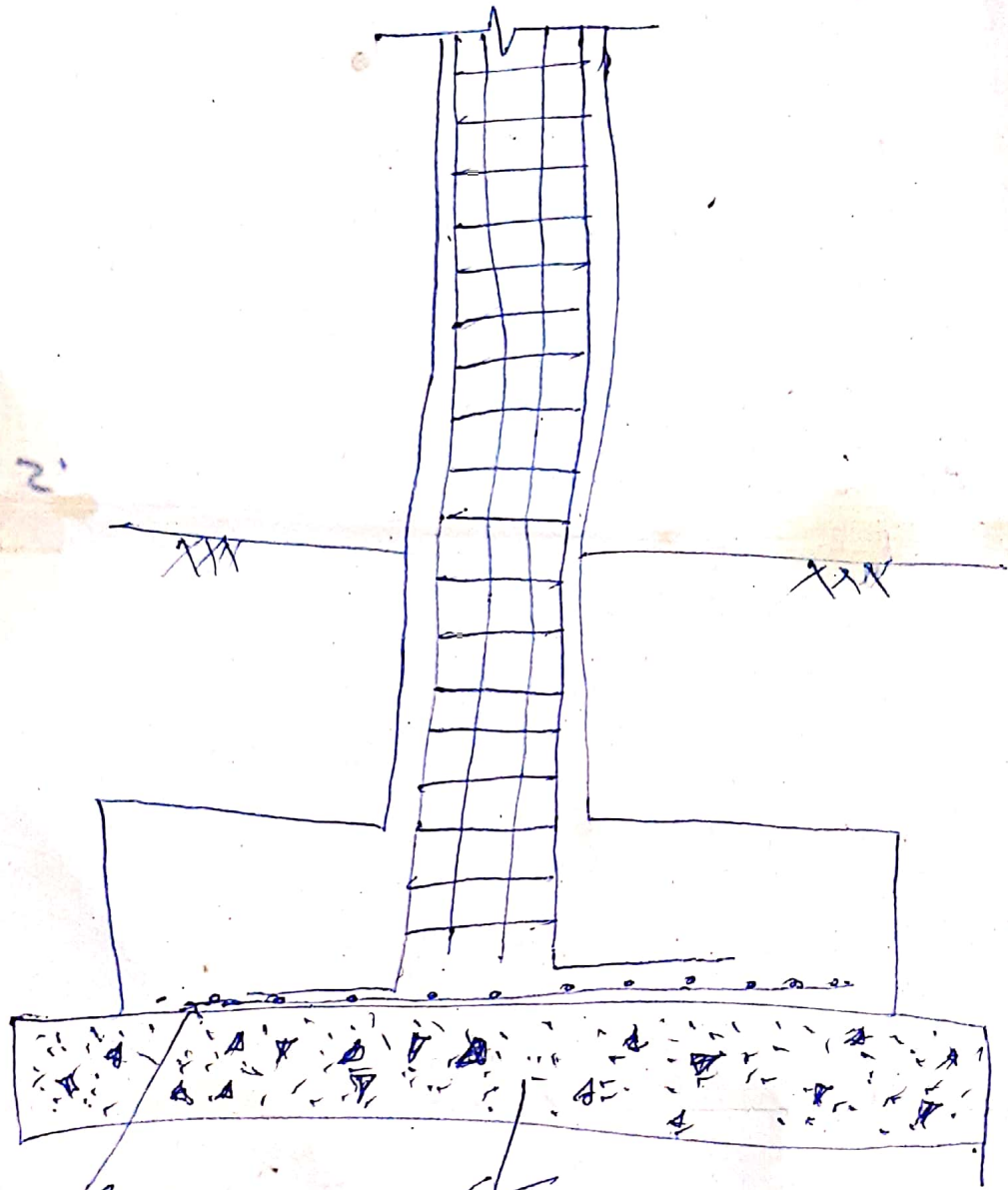
It is satisfy.

$$\therefore \text{size of the footing} = 2.5 \text{ m} \times 2.5 \text{ m}$$

$$\text{Depth of the footing} = 400 \text{ mm}$$

— x —

2' for 1/2" dia



Cover  
60mm

P.C.C  
Con.

18 > 0



# Design of footing (S.D. 11)

Q1 <sup>(2025)</sup> A R.C.C Column  $500 \times 500$  mm in section carries an axial load of  $600$  kN. Design an isolated footing of uniform thickness for the column. The safe bearing capacity of soil is  $120$  kN/m<sup>2</sup>. Use M15 concrete and mild steel.

soln: Column size =  $500$  mm  $\times$   $500$  mm.

Load =  $600$  kN.

Foundation load =  $10\%$  of Column load

Total load =  $600 + 60 = 660$  kN.

S.B.C =  $120$  kN/m<sup>2</sup>.

$f_{ck} = 15$  N/mm<sup>2</sup>.

$f_y = 250$  N/mm<sup>2</sup>.

Ultimate load =  $1.5 \times 660 = 990$  kN.

$A_{\text{required}} = \frac{\text{Ultimate load}}{\text{S.B.C}} = \frac{990}{120} = 8.25 \text{ m}^2$ .

Let  $B = 3$  m,  $L = 3$  m.

$A_{\text{provided}} = 3 \times 3 = 9 \text{ m}^2$ .

Upward pressure =  $\frac{\text{Column load}}{\text{Area provided}}$

$$= \frac{1.5 \times 600}{9} = 100 \text{ kN/m}^2 < \text{S.B.C.}$$

Bending Moment calculation:

$$M_{xx} = M_{yy} = \underset{\substack{\uparrow \\ \text{upward} \\ \text{pressure}}}{q} \cdot B \times \frac{(B-b)^2}{8}$$

$$= 100 \times 3 \times \frac{(3-0.5)^2}{8} = 234.375 \text{ kNm.}$$

P.T.O



## Depth Calculation:

(2)

$$M_{lim} = 0.36 f_{ck} \frac{x_{max}}{d} \left( 1 - 0.42 \frac{x_{max}}{d} \right) b d^2$$

$$= 234.375 \times 10^6 = 0.36 \times 15 \times 0.46 \left[ 1 - (0.42 \times 0.46) \right] \times 3000$$

$$\Rightarrow d = 197.44 \text{ mm} = 300 \text{ mm}$$

$$D = d + \text{clear cover}$$

$$= 300 + 50 = 350 \text{ mm}$$

## Steel Calculation.

$$A_{stx} = A_{sty}$$

$$\frac{M_u}{b d^2} = \frac{237.375 \times 10^6}{3000 \times (300)^2} = 0.86$$

$$P_t = 0.450$$

$$A_{st} = \frac{0.450}{100} \times 3000 \times 300 = 4050 \text{ mm}^2$$

Provide 20 mm of 13 nos.

$$\text{So } A_{\text{provided}} = 4080 \text{ mm}^2$$

Check for one way shear.

Critical section 'd' from the face of the column

$$i_v \leq k i_c$$

$$i_v = \frac{V_u}{b d}$$

$$V_u = q \times L \times \left( \frac{L-d}{2} - d \right)$$

$$= 100 \times 3 \times \left( \frac{3 - 0.5 - 0.3}{2} \right)$$

$$= 285 \text{ kN}$$

$$i_v = \frac{V_u}{bd} = \frac{2.85 \times 10^3}{3000 \times 300} = 0.31$$

(3)

Here  $k = 1$ .

$$P_t = 0.450$$

$$i_c = 0.36 \text{ N/mm}^2$$

$$K i_c = 1 \times 0.36 = 0.36 \text{ N/mm}^2$$

$i_v < K i_c$ , it is satisfied.

Section is safe.

What is the

1) Min value of the thickness of a stepped footing at the edge when the footing rests on a normal soil?

sol<sup>n</sup>: 150 mm is the min value of the thickness of a stepped footing at the edge when the footing rests on a normal soil.

Q2: what is the minimum depth of foundation for a soil with S.B.C of  $150 \text{ kN/m}^2$ , unit wt of  $20 \text{ kN/m}^3$  and  $\phi = 30^\circ$ ?

$$\begin{aligned} \text{sol}^n: \quad D &= \frac{\text{S.B.C}}{20} \left[ \frac{1 - \sin \phi}{1 + \sin \phi} \right]^2 \\ &= \frac{150}{20} \left( \frac{1 - \sin 30^\circ}{1 + \sin 30^\circ} \right)^2 \\ &= 7.5 \times \left( \frac{1}{2} \times \frac{2}{3} \right)^2 \\ &= 0.833 \text{ m} \end{aligned}$$

P.T.O

Q3 : where the critical sections for <sup>(1)</sup> computing max bending moment in each of the following cases lie (a) footing supporting a column (b) footing supporting a masonry wall.

Ans : (a)  $d/2$  from the face of the column where footing supporting a column.

(b)  $d$  from face of the wall where footing supporting a masonry wall.

Q4 : Differentiate bet<sup>n</sup> an isolated footing and a strip footing.

Ans : An individual footing under a single column is known as an isolated footing. where good soil and sufficient area is available, these footings are economical.

If a number of footings in a line are to be combined, a strip footing is used. Differential ~~footing~~ settlement can be minimized by using such footing.

S.D-II.

Design of footing

VI<sup>th</sup> Sem (Civ)

Q2: Design uniform depth R.C.C footing for a masonry wall 25cm thick subjected to a load of 100 kN/m inclusive of self weight. The S.B.C of soil is 120 kN/m<sup>2</sup>.

Assuming M<sub>20</sub> Concrete and Fe415

Design Constants:

$$R_c = 0.289, I_c = 0.904, \text{ and } \rho_c = 0.914.$$

$$\begin{aligned} \text{Width } B \text{ of footing} &= \frac{\text{Load}}{\text{S.B.C of soil}} \\ &= \frac{100 \text{ kN/m}}{120 \text{ kN/m}^2} \\ &= 0.833 \text{ m} \end{aligned}$$

$$\text{Adopt } B = 0.9 \text{ m}$$

$$\begin{aligned} \text{Net upward pressure } P_0 &= \frac{\text{Load}}{B} \\ &= \frac{100}{0.9} = 111.11 \text{ kN} \\ &= \underline{\underline{111.11 \text{ kN}}} \end{aligned}$$

Design section:

Max bending moment occurs at section X-X distance  $b/4$  from the centre of the wall and its magnitude is given by.

$$\begin{aligned} M &= \frac{P_0}{8} (B-b) (B - b/4) \times 10^6 \text{ Nmm} \\ &= \frac{111.11}{8} (0.9 - 0.25) \times 10^6 \text{ Nmm} \\ &= 7.56 \times 10^6 \text{ Nmm} \end{aligned}$$

*b = thickness of masonry wall.*

P.T.O

$$d = \sqrt{\frac{M}{\rho_c}}$$

where  $\rho_c$  is <sup>(2)</sup>  
design constant  
= 0.914.

$$= \sqrt{\frac{7.56 \times 10^6}{1000 \times 0.914}} = 90.95 \text{ mm}$$

Provide Total Depth  $D = 160 \text{ mm}$   
Cover = 60 mm  
to the centre of the steel

$$\therefore \text{Available depth } d = 160 - 60 = 100 \text{ mm}$$

Check for shear:

For balanced section.

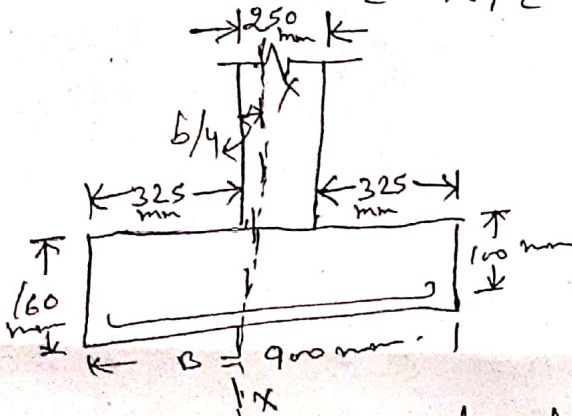
$P = 0.44\%$  for  $M_{20}$  concrete and Fe 415 steel.

$$Z = 0.28 \text{ N/mm}^2$$

$$K = 1.30, \quad M_c = 0.28$$

Hence, permissible shear stress

$$= k M_c = 1.3 \times 0.28 = 0.364 \text{ N/mm}^2$$



Distance of critical section from edge of footing

$$= \frac{1}{2} (B - d) - d$$

$$= \frac{1}{2} (0.9 - 0.25) - 0.100$$

$$= 0.225 \text{ m}$$

$$V_u = 106000 \times 0.225 = 23850 \text{ N/m}$$

$$V_u = P_0 \times B \left[ \frac{1}{2} (B - d) - d \right], \quad P_0 = \frac{\text{Load} \cdot T.O}{R_x R_y}$$

$$M_v = \frac{V}{bd} = \frac{23850}{1000 \times 100} = 0.2385 \text{ N/mm}^2$$

Less than  
Permissible shear  
stress.  
Hence safe

Design of reinforcement.

$$A_{st} = \frac{M}{\sigma_s \times jd}$$

$$= \frac{7.56 \times 10^6}{230 \times 0.904 \times 100} = 363.6 \text{ mm}^2$$

→ c/c (constant)

Using 12 mm  $\phi$  bars.

$$\text{Area of steel bar} = \frac{\pi}{4} \times 12^2 = 113 \text{ mm}^2$$

$$\text{Spacing } S = \frac{1000 \times \text{Area of one bar}}{\text{Area of steel } A_{st}}$$

$$= \frac{1000 \times 113}{364} = 310 \text{ mm}$$

Provide 12 mm  $\phi$  bar @ 300 mm c/c.

Area of longitudinal reinforcement  
= 0.12% of area of cross-section.

$$= \frac{0.12}{100} \times 100 \times 160$$

↓  
Total depth.

$$= 50.26 \text{ mm}^2$$

$$\text{Spacing} = \frac{1000 \times 50.26}{192} = 261 \text{ mm}$$

Provide 8 mm  $\phi$  bars @ 250 mm c/c.

Timber beams are wooden structural members which support load primarily by ~~its~~ its internal resistance to bending. Wooden beams usually consist of single piece of rectangular or built up section though it may be of square or circular section.

Effective span: of simply supported beams shall be taken as distance from face to face of supports plus one-half of the required length of bearings at each end.

Width: Minimum width of beams should not be less than 5 cm or  $1/50$  of the span whichever is greater.

Depth: of the beam ~~should~~ should not be taken more than three times its width.

Bending strength:

pure bending equation for flexural strength.

$$M = f_{ab} \times Z$$

where  $f_{ab}$  - Calculated bending stress.

$Z$  = Elastic section modulus.

$$f_{ab} \leq f_b$$

where  $f_b$  = permissible bending stress.

Form factors:

(a) Rectangular sections: ( $D > 300 \text{ mm}$ ).

For different depth of beams the form factor  $k_3$  is taken as

$$k_3 = 0.81 \left( \frac{D^2 + 89400}{D^2 + 55000} \right)$$

where  $D$  is the depth of beam in mm.

Teacher's Signature: \_\_\_\_\_

(b) Box beams & I-beams:

(2)

Form factor  $K_u$  is

$$K_u = 0.8 + 0.8 \gamma \left( \frac{D^2 + 89400}{D^2 + 55000} - 1 \right)$$

where  $\gamma = p_1^2 (6 - 8p_1 + 3p_1^2) (1 - q_1) + q_1$

$p_1$  = Ratio of thickness of Compression flange to the depth ( $d_1/d$ ) of the beam.

$q_1$  = Ratio of total thickness of webs to the overall width of beam.

(c) Solid circular cross-section: For solid circular cross-section, the form factor  $K_5$  is taken as 1.18.

(d) Square cross-section: For square cross-section, where the load is in the direction of diagonal, the form factor  $K_6$  is taken as 1.414.

Shear in beams: Beams are checked for Max horizontal shear  $H$ . Max shear stress can be found by

(a) Rectangular beams:

$$H = \frac{3V}{2bD}$$

(b) Notched beams, with tension notch at the supports,  $H = \frac{3VD}{2bD^2}$

(c) Notched at upper (Compression) face where  $e > D$ ,  $H = \frac{3V}{2bD_1}$



Shear in beams.

(1) Notched at upper (compression) face where

$$H = \frac{3V}{2b \left[ D - \left( \frac{D_2}{D} \right) e \right]}$$

where  $V$  = Shear force at the section.

$b$  = width of the beam.

$D$  = Depth of beam.

$D_1$  = Depth of beam at notch.

$D_2$  = Depth of the notch.

$e$  = length of notch

Maximum Shear force "  $V$  is calculated as.

(i) for concentrated load  $C$  at a distance  $x$  from the support,

$$V = \frac{10C(1-x)(x/D)^2}{9L[2 + (x/D)^2]}$$

where  $C$  = concentrated load.

$L$  = span of beam.

$x$  = Distance from reaction to load.

(ii) for Uniformly distributed load :

$$V = \frac{W}{2} \left( \frac{1 - 2D}{L} \right) \text{ or } w \left( \frac{L}{2} - D \right)$$

where  $L$  = span of the beam.

$W = u.d.l$  per unit length.

Teacher's Signature: \_\_\_\_\_

P. 7.0

Deflection: The deflection in case of flexural members ~~that~~ should not exceed  $1/240$  of the span. The deflection in case of Cantilevers should not exceed  $1/150$  of the freely hanging length. (4)

$$\text{Deflection} = \frac{k W l^3}{E I} \quad \text{or} \quad k \cdot \frac{W l^3}{E I} = k \cdot \frac{w l^4}{E I}$$

where  ~~$k = \frac{1}{48}$~~

$k = \frac{1}{48}$  for simply supported beams with central concentrated load.

$k = \frac{5}{384}$  for simply supported beam with uniformly distributed load.

$W = \text{total load} = w l$

$l = \text{span of beam}$

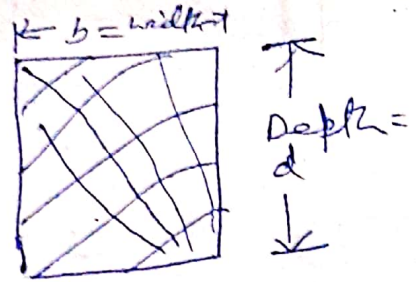
Timber Structures

Design of simple timber structural elements in

Flexure :

Beams : are defined as the structural members which support the load primarily by its internal resistance to bending.

Wooden beam usually consists of a single piece of rectangular section.



Builtup wooden beams : are formed by connecting a number of smaller beams together by bolts, screws, or spikes.

Following form factors are applied to the bending stress for the following cross-sections of the beams as per IS: 883-1994.

(a) Rectangular section : for rectangular section for different depth of beams the

Form Factor  $k_3 = 0.81 \left( \frac{D^2 + 89400}{D^2 + 55000} \right)$   
 where  $D =$  Depth of beam.  
 $k_3$  shall be applied for beams having depth less than or equal to 300 mm.

(b) Box beams and I-beams :

Form Factor

$$k_4 = 0.5 + 0.87 \left( \frac{D^2 + 89400}{D^2 + 55000} - 1 \right)$$

where  $\gamma = P_1^2 (6 - 8P_1 + 3P_1^2) (1 - \gamma_1) + \gamma_1$

(\*)  $P_1 =$  Ratio of thickness of the compression flange to the depth of the beam

$a_1$  = Ratio of total thickness of web to the overall width of the beam.

Check for shear:

Max horizontal shear stress at neutral axis  $f_{sh} = \frac{V \cdot Q}{I \cdot b}$

where  $f_{sh}$  = Horizontal shear stress in beam in  $N/mm^2$ .

$V$  = Vertical shear in  $N$ .

$b$  = width of beam in  $mm$ .

$D$  = Depth of beam section in  $mm$ .

$I$  = Moment of Inertia of section in  $mm^4$ .

$Q$  = statical moment of area, in  $mm^3$ .

Max shear stress for

① Rectangular beams

$$f_{sh} = \frac{3}{2} \left( \frac{V}{b \cdot D} \right)$$

② Notched beams: Notched at tension face at the support.

$$f_{sh} = \frac{3}{2} \cdot \left[ \frac{V \cdot D}{b \cdot D_1^2} \right] \text{ where } D_1 = \text{Depth of beam at notch in } mm$$

③ Notched at Compression face -  $D_2 = \text{Depth of beam in } mm$

$$f_{sh} = \frac{3}{2} \left[ \frac{V}{b \cdot D_2} \right]$$

For rectangular beams

$$Q = \frac{1}{2} b \cdot D^2 \quad \text{and} \quad I = \frac{1}{12} b \cdot D^3$$

$$f_{sh} = \frac{V \cdot Q}{I \cdot b}$$

$$f_{sh} = \frac{3 \cdot V}{2 \cdot b \cdot D}$$

P.T.O

Value of  $V$  can be calculated from the following formula.

For Concentrated

$$\text{loads } V = \frac{100(l-x)(x/D)^2}{91(2+(x/D))^2}$$

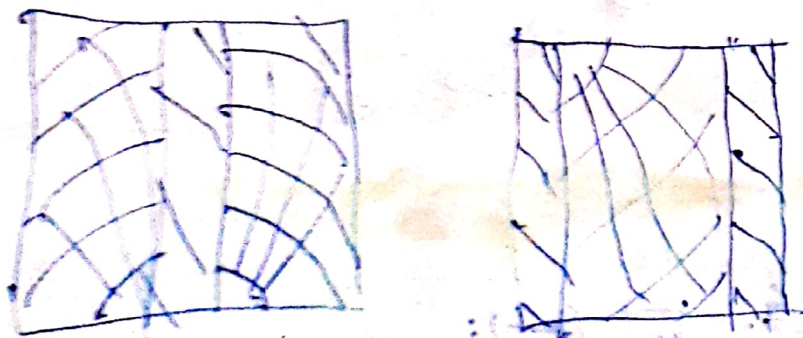
where  $c$  = concentrated load in  $N$ ,  
 $l$  = span of beam in  $m$ .

$x$  = Distance from reaction to load in  $m$ .

For Uniformly distributed loads

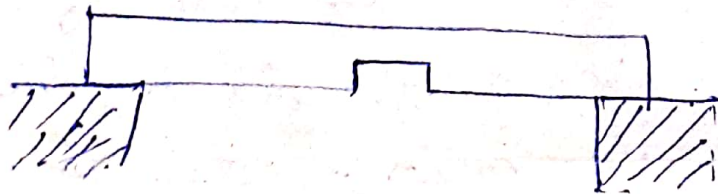
$$V = \frac{W}{2} \left(1 - \frac{2D}{l}\right) \sqrt{\quad}$$

Fitched beams : consists of wooden beams and steel beams joined together by means of bolts or screws. When the timber sections are joined together with steel plates, then the deformations in the fibres of timber and steel in fitched beams are equal. The modulus of elasticity of a material is defined as stress divided by unit deformation.

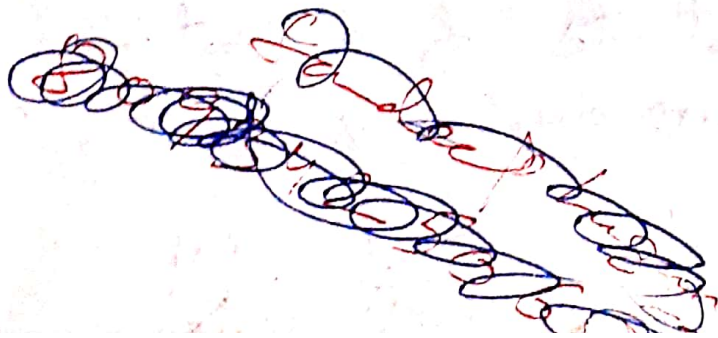


Fitched beams.

(1)  
Notched beams : When a groove is cut either at the ends or at the middle of span or anywhere in bet<sup>n</sup> support in the timber beams, then beams are known as Notched beams. The beams are cut or notched at the ends to reduced the depth of floors. Sometimes, the beams are notched to increase the room clearance. The beams are notched at the middle or any where bet<sup>n</sup> the supports to provide space for pipes or support to other beams and frames. The cross-sectional area of beams at notches are reduced.



Notched beams



Q1  
20086, A deodar timber beam carries a uniformly distributed load of  $18 \text{ kN/m}$  inclusive of self wt of the beam. The beam is simply supported at both ends. The clear span of beam is  $5 \text{ m}$ . Design the timber beam. The allowable bending stress for deodar wood is  $10.2 \text{ N/mm}^2$ .

Ans Provide  $250 \text{ mm}$  bearing at each end effective span  $= 5.25 \text{ m}$ .

$$B.M = \frac{wl^2}{8}$$

$$\text{Max B.M} = \frac{18 \times 5.25^2}{8} = 62.015 \text{ kNm}$$

Permissible bending stress for deodar wood

$$\text{Section Modulus (Z)} = \frac{M}{\sigma_{bc}} = \frac{62.015 \times 10^6}{10.2} = 6079963.2 \text{ mm}^3$$

$$\text{Breadth of beam} = \frac{\text{span}}{25} = \frac{5250}{25} = 210 \text{ mm}$$

$$\text{Section Modulus} = \frac{bd^2}{6} = 210 \times \frac{d^2}{6} = 6079963.2$$

$$\Rightarrow d^2 = 173713.23$$

$$d = 416.79 \text{ mm}$$

Provide depth  $= 420 \text{ mm}$ .

According to code depth shall not be taken more than  $3 \times b$  without lateral stiffeners.

(2)

$$3b = 3 \times 210 = 630 \text{ mm}$$

$$420 < 630 \text{ mm}$$

As depth is more than 300 mm form factor.

$$\begin{aligned}
 K_3 &= 0.81 \left( \frac{D^2 + 89400}{D^2 + 55000} \right) \\
 &= 0.81 \left( \frac{420^2 + 89400}{420^2 + 55000} \right) \\
 &= 0.81 \left( \frac{265800}{231400} \right) = 0.81 \times 1.1486 \\
 &= 0.93
 \end{aligned}$$

$$\text{Bending stress} = \frac{M}{K_3 Z} \rightarrow \frac{bd^2}{6}$$

$$= \frac{62.015 \times 10^6 \times 6}{0.93 \times 210 \times 420^2}$$

$$= 10.80 > 10.2 \text{ N/mm}^2$$

(Not safe  $\rightarrow$ )

$$\therefore K_3 = 0.81 \left( \frac{450^2 + 89400}{450^2 + 55000} \right)$$

$$= 0.81 \times \frac{291900}{257500}$$

$$= 0.81 \times 1.1335 = 0.918$$

$$\text{Bending stress} = \frac{62.015 \times 10^6 \times 6}{0.918 \times 210 \times 450^2}$$

$$= 9.53 \text{ N/mm}^2 < 10.2 \text{ N/mm}^2$$

Check for shear:

$$V = \frac{W}{2} \left( 1 - \frac{2d}{l} \right)$$

P.T.O



$$= \frac{15 \times 10^3 \times 5.25}{2} \left( 1 - \frac{2 \times 450}{525} \right) \quad (3)$$

Horizontal shear stress

$$H = \frac{3}{2} \frac{V}{bd}$$

$$= \frac{3 \times 32625}{2 \times 210 \times 450}$$

(where  $0.7 = 0.5178 < 0.7$  permissible shear for double)

$$\text{Allowable defn} = \frac{l}{240} = \frac{5250}{240}$$

$$= 21.875 \text{ mm.}$$

$$\text{Actual defn} = \frac{5WL^3}{384EI}$$

$$= \frac{5 \times (15 \times 10^3 \times 5.25) \times 5250^3}{384 \times 9500 \times 210 \times 450^3}$$

$$= 9.794 \text{ mm} < 21.875$$

$$I = 210 \times 450^3$$

Q2: A simply supported timber beam carries a total uniformly distributed load of  $50 \text{ kN}$  inclusive of self weight. The effective span of beam is  $8 \text{ m}$ . The timber beam is made of 'sal' wood.  $300 \text{ mm} \times 50 \text{ mm}$  planks are only available. Design a built-up beam.

P.T.O

~~10~~ ~~10~~ ~~10~~ ~~10~~

Effective span :

The strength of a built-up timber beam is equal to sum of the strengths of each element of the built-up beam.  
Total uniformly distributed load including self-weight = 50 kN.  
Effective span = 8m.

Max Bending moment :

$$M = \frac{W l^2}{8} = \frac{50 \times 8000^2}{8 \times 1000} = 50 \text{ kNm}$$

From IS: 883-1970. Safe working stress in bending for wide location on standard grade sal wood = 16.8 N/mm<sup>2</sup>.

Sectional Modulus required

$$Z = \frac{50 \times 6}{16.8} = 2976.19 \times 10^3 \text{ mm}^3$$

Section Modulus of 300 mm x 50 mm plank

$$Z_1 = \frac{1}{6} \times 50 \times 300 \times 300 = 750 \times 10^3 \text{ mm}^3$$

Number of planks required  $\left( \frac{1}{6} b d^2 \right)$

$$\frac{Z}{Z_1} = \frac{2976.19 \times 10^3}{750 \times 10^3} = 3.968 \approx 4$$

Provide 4 planks of 300 mm x 50 mm. The planks are bolted together. The bolts are spaced 1.0 m apart. (Less than 4 times the depth of the beam).

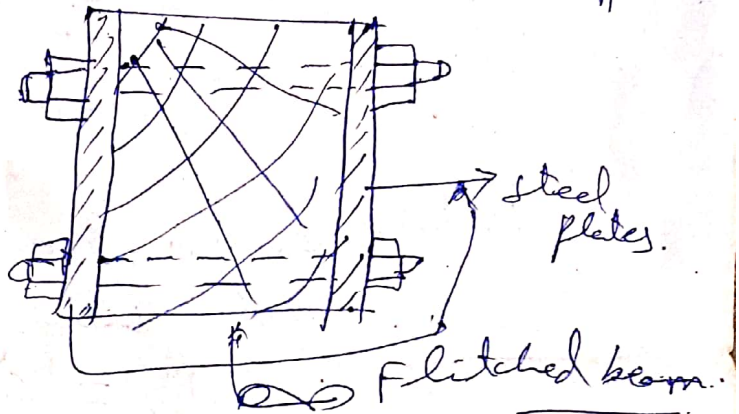
Bending (Flexural) and shear strength of Timber beams.

Q1: A beam is simply supported at its both the ends. The effective span of beam is 6 m. It consists of 200 mm x 300 mm teak wood with 300 mm x 12 mm steel plates bolted to its sides. Determine the safe uniformly distributed load, which the beam will support.

Modulus of section of two steel plates.

$$Z = 2 \times \frac{1}{6} \times 12 \times 300^2$$

$$= 360 \times 10^3 \text{ mm}^3$$



It is assumed that the beam is laterally supported. For ~~lateral~~ laterally supported beam allowable stress in bending at the extreme fibre in steel,  $f_b = 0.66 \times 250 = 165 \text{ N/mm}^2$ .

Moment of resistance of steel plates.

$$M = f_b \cdot Z = 165 \times \frac{360 \times 10^3}{1000 \times 1000}$$

Load supported by beam:  $= 59.4 \text{ kN-m}$ .

Let  $W$  be the u.d.l supported by the steel plates

$$\therefore \frac{WL}{8} = 59.4 \times 10^6$$

$$\therefore W = \frac{59.4 \times 10^6 \times 8}{6000 \times 1000} = 79.2 \text{ kN}$$

For teak wood: From IS - 883 - 1970.

$$E_w = 9600 \text{ N/mm}^2$$

for ~~wood~~  
teak wood.

$$E_s \text{ for steel} = 2.06 \times 10^5 \text{ N/mm}^2$$

stress in the extreme fibre of wood.

$$f_w = f \times \frac{E_w}{E_s} = 165 \times \frac{9600}{2.06 \times 10^5}$$

$$= 7.689 \text{ N/mm}^2 < 14 \text{ N/mm}^2$$

Modulus of section of wooden

$$\text{beam } z_1 = \frac{1}{6} \times 200 \times 300^2$$
$$= 3000 \times 10^3 \text{ mm}^3$$

Modulus of resistance of  
wooden beam

$$M_1 = f_w \times z_1$$

$$= 7.689 \times 3000 \times 10^3$$

$$\frac{\quad}{1000 \times 1000} = 23.067 \text{ kNm}$$

Let  $W_1$  be the U.d.l supported by wooden beam.

$$\frac{W_1 L}{8} = 23.067 \times 10^6 = M_1$$

$$\text{or } W_1 = \frac{23.067 \times 10^6 \times 8}{6000 \times 1000}$$

$$= 30.756 \text{ kN}$$

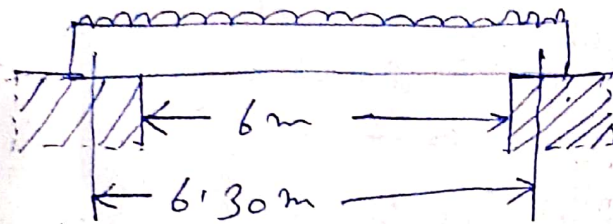
The U.d.l inclusive of the self wt of  
fitched beam =  $W + W_1$

$$= 79.2 + 30.756$$

Q1) <sup>2008</sup> A timber beam having a clear span of 6.0 m carries a uniformly distributed load of 15 kN/m including the self wt of beam.

<sup>2010(S)</sup> Assuming the beam to be made of Deodar wood, design the beam.

Ans: clear span = 6m.



Assume width of the bearing at each end = 300mm.

$$\text{Effective span of beam} = 6 + \frac{0.3}{2} + \frac{0.3}{2} = 6.3 \text{ m.}$$

Max Bending Moment

$$M = \frac{wl^2}{8} = \frac{15 \times 6.3^2}{8} = 74.42 \text{ kNm.}$$

Form factor for Rectangular section  $K_3 = 0.81 \left( \frac{D^2 + 89400}{D^2 + 55000} \right)$

Assume depth of beam = 400 mm.

$$\therefore K_3 = 0.81 \left( \frac{400^2 + 89400}{400^2 + 55000} \right) = 0.9396$$

The max allowable bending stress  $\sigma_b$  of Deodar wood is 10.2 N/mm<sup>2</sup> (since D > 300mm).

Maximum allowable bending stress,  $\sigma_b = 0.9396 \times 10.2 = 9.584 \text{ N/mm}^2$

$K_3 = 9.584 \text{ N/mm}^2$

## Section Modulus:

$$Z_{req} = \frac{\text{Max B.M}}{\text{Max allowable bending stress}}$$

$$= \frac{74.42 \times 1000 \times 1000 \xrightarrow{\text{KN to N}} \xrightarrow{\text{m to mm}}}{9.584}$$

the beam is considered laterally supported if  $b > \frac{d}{50}$  and  $b > \frac{d}{3}$

width of beam required

$$b = \frac{1}{50} \times 6.3 \times 10^3 = 126 \text{ mm}$$

Adopt  $b = 250 \text{ mm}$   
 $d = 400 \text{ mm}$

The Modulus of section of rectangular beam

$$\frac{1}{6} b d^2 = \frac{1}{6} \times 250 \times d^2 = 41.67 d^2$$

$$\therefore 41.67 d^2 = 776.5 \times 10^4$$

$$d = 431$$

$$\text{Adopt } d = 400 \text{ mm}, \frac{d}{3} = \frac{400}{3} = 133.3 \text{ mm} < b$$

Not required

## Check for shear:

Max shear force at the edge of the support

$$V = W \left( \frac{L}{2} - D \right) = 15 \times \left( \frac{6.3}{2} - 0.4 \right)$$

$$= 41.25 \text{ kN}$$

Max shear stress in the beam

$$f_{sh} = \frac{3}{2} \left( \frac{V}{b \cdot d} \right)$$

$$= \frac{3}{2} \times \left( \frac{41.25 \times 10^3}{250 \times 400} \right)$$

$$= 0.62 \text{ N/mm}^2$$

Permissible shear stress =  $0.9 \text{ N/mm}^2$

check for deflection:

$$\text{Max deflection } \delta_{\text{max}} = \frac{5}{384} \times \frac{Wl^4}{EI}$$

$$\delta_{\text{max}} = \frac{5}{384} \frac{Wl^3}{EI}$$
$$= \frac{5}{384} \frac{wl \cdot l^3}{EI}$$
$$= \frac{5}{384} \frac{wl^4}{EI}$$

$$E = 9.5 \times 10^3 \text{ N/mm}^2$$

$$I = \frac{bd^3}{12}$$

$$= \frac{250 \times 400^3}{12} = 1333 \times 10^6 \text{ mm}^4$$

$$\therefore \delta_{\text{max}} = \frac{5}{384} \times \left[ \frac{75 \times (6300)^4}{9.5 \times 10^3 \times 1333 \times 10^6} \right]$$
$$= 24.3 \text{ mm}$$

$$\text{Allowable deflection} = \frac{6300}{250} = 25.2 \text{ mm}$$

$\therefore \delta_{\text{max}} < \text{Allowable deflection}$ , hence ok

check for bearing:

$$\text{Reaction at the support} = \frac{Wl}{2}$$

$$= \frac{15 \times 6.3}{2}$$

$$= 47.25 \text{ kN}$$

Bearing stress at the

$$\text{support} = \frac{\text{Reaction at the support}}{b \times d}$$

$$= \frac{47.25}{250 \times 400} = 0.47 \text{ N/mm}^2$$

$$\text{Safe working stress} = 2.6 \text{ N/mm}^2$$

$\rightarrow \text{const.}$

$\therefore \text{Bearing working stress} < \text{safe working bearing stress}$

Hence satisfactory.

$\therefore$  Provide beam of width = 250 mm and depth = 400 mm

Q1 The effective length of compression flange of simply supported beam MB 500 @ 0.869 kN/m is 8m. Determine the safe uniformly distributed load per metre length which can be placed over the beam having an effective span of 8 metres. Adopt max permissible stresses as per IS 800 - 1984. The ends of beam are restrained against rotation at the bearing.

Soln: Permissible bending stress:

MB 500 @ 0.869 kN/m: Effective span = 8m.

Effective length of compression flange = 8m.

From steel table:

Section modulus of beam

$$Z = 1808.7 \times 10^3 \text{ mm}^3.$$

mean thickness of compression flange  $t_f = T = 17.2 \text{ mm}$ .

Thickness of web,  $t_w = 10.2 \text{ mm}$ .

Assumed yield stress  $f_y = 250 \text{ N/mm}^2$ .

$$\text{Ratio } \frac{T}{t_w} = \frac{17.2}{10.2} = 1.686 < 2.0.$$

$$\text{Ratio } \frac{d_e}{t_w} = \frac{L - 2k_2}{t_w} = \left( \frac{500 - 2 \times 37.95}{10.2} \right).$$

$$\left( \frac{424.1}{10.2} \right) = 41.578 < 85.$$

$$\text{Ratio } \left( \frac{D}{T} \right) = \frac{500}{17.2} = 29.07.$$

$$\text{Ratio } \frac{L}{r_y} = \frac{0.7 \times 8 \times 1000}{35.2} = 159.1.$$

From IS 800 - 1984 from Table

The max permissible bending stress

$$\sigma_{bc} = 88.566 \text{ N/mm}^2$$

Teacher's Signature: \_\_\_\_\_



Moment of ~~resistance~~ resistance of beam

$$M \cdot R = \sigma_{bc} \times Z \\ = \frac{88.566 \times 1808.7 \times 10^3}{1000 \times 1000} = 160.189 \text{ m-kN}$$

$$\therefore \text{Max B.M} = M \cdot R = 160.189 \text{ m-kN}$$

Load supported over beam :

Effective span of beam is 8m.

Let  $w$  be the uniformly distributed load per metre length.

$$\therefore \text{Max B.M} = \frac{w l^2}{8} \\ = \frac{w \times 8 \times 8 \times 1000}{8} = 160.189$$

$$\therefore w = \frac{160.189 \times 8}{8 \times 8 \times 1000} \times 1000 \\ = 20.02 \text{ kN/m}$$

Self wt of the beam is 0.869 kN/m.

$$\therefore \text{safe u.d.l} = 20.02 - 0.869 \\ = 19.15 \text{ kN/m}$$